HILLIARD CITY SCHOOLS

K-12
Mathematics

Course of Study

Hilliard City Schools

David Stewart, Superintendent
Mike McDonough, Deputy Superintendent
Jill Abraham, Assistant Superintendent
Cori Kindl, Executive Director of Curriculum

Hilliard City School District's Board of Education Adoption:

Mathematics Course of Study

Table of Contents

Forward and Acknowledgement

The Hilliard City School District would like to acknowledge and thank the following staff who served on the K-12 Mathematics Curriculum Revision Committee. This committee consisted of staff from all grade levels, buildings, disciplines and supports. Through their professional development, content knowledge, instructional experience, and knowledge of students, the committee revised the K-12 Mathematics Course of Study to incorporate the Ohio Learning Standards for Mathematics, evidence-based research, strong instructional strategies, and assessment practices. Thank you for the time you have given and the work you completed to design this Course of Study for our teachers and students..

Dr. Kimberle Kembitzky - Teacher Leader: Secondary Mathematics	Central Office
Betsy Long - Teacher Leader: Elementary Mathematics	Central Office
Adriana Kruse - Kindergarten	Alton Darby
Alli Palguta - Kindergarten	Britton
Molly Dickmann - Kindergarten	Ridgewood
Kim McCreary - First Grade	Scioto Darby
Tabatha Stover - First Grade	Horizon
Courtney Brown - First Grade	Brown
Mallory McElhose - Second Grade	JW Reason
Denise Thurston - Second Grade	Washington
Brittany Sheedy - Second Grade	Horizon
Kristen Rigsby - Third Grade	Norwich
Liz Ernst - Third Grade	Avery
Nancy Ley - Third Grade	Beacon
Jillian Kalb - Fourth Grade	Norwich
Nicole Burger - Fourth Grade	Avery
Cecilia Maziarz - Fourth Grade	JW Reason
Katie Putrino - Fifth Grade	Hoffman Trails
Tricia Holtkamp - Fifth Grade	Darby Creek
Morgan Hall - Fifth Grade	Ridgewood
Nicole Brown - Math Support Teacher	Horizon
Sara Firestone - Math Support Teacher	Alton Darby
Claudette Mullins - Math Support Teacher	Crossing
Jennifer Bickley - Intervention Teacher	Scioto Darby
Brenda Boring - Intervention Teacher	Norwich
Jodi Sierer - Intervention Teacher	Britton
Whitney Jeckavitch - Intervention Teacher	Avery
Robin Schroot - EL Teacher	Avery
Anna Lepper - EL Teacher	Darby Creek
Josh Bayman - Intervention Specialist	Alton Darby
Carrie Darr - Intervention Specialist	Norwich
Chuck D’Andrea - Innovation Discovery Specialist	Brown
Rachel Nelli - Gifted Intervention Specialist	Avery, Crossing
Kevin Landon - Principal	Avery
Kate Miller - Principal	Brown
Kevin Buchman - Principal	Ridgewood
Stephanie Borlaza - Principal	Britton
Holly Meister - Principal	Scioto Darby
Cindy Teske - Principal	Darby Creek

Monica Campana - Principal	Washington
Kristin Myers - Sixth Grade	Station
Allison Williams - Sixth Grade	Station
Erica Klenotic - Sixth Grade	Tharp
Alicia Tremayne - Seventh Grade	Heritage MS
Elaine Lindenbaum - Seventh Grade	Heritage MS
Chase Hampton - Seventh Grade	ILC/HUB
Jeff Salyer - Seventh Grade	Weaver MS
Tenise Hawk - Eighth Grade	Heritage MS
Heather Moody - Eighth Grade	Memorial MS
Rob Daniels - Eighth Grade	Weaver MS
Laura Tornes - Algebra 1	Heritage MS
Matthew Koob - Algebra 1	Memorial MS
Brett Miller - Algebra 1	Darby HS
Cami Kowalski - Algebra 1	Bradley HS
Patricia Bergstrom - Algebra 1	Bradley HS
Heather Usner - Algebra 1	Davidson HS
Angela Miyamoto - Geometry	Heritage MS
Scott Allen - Geometry	Davidson HS
Andrew Ozbolt - Geometry	Darby HS
Melissa Gmerek - Geometry	Bradley HS
Emily Wilson - Geometry	Davidson HS
Scott Dietrich - Geometry	Darby HS
Amy Melsop - Algebra 2	Davidson HS
Adam Rachuba - Algebra 2	Davidson HS
Brent Bower - Algebra 2	Online Academy
Rebecca Biller - Algebra 2	Darby HS
Jackie Fowler - Algebra 3	Darby HS
James Desenberg - Algebra 3	Bradley HS
Rich Chute - PreCalculus	Darby HS
Kristen Horne - PreCalculus	Bradley HS
Katie Sims - Statistics/AP Statistics	Darby HS
Kevin Moody - Statistics/AP Statistics	Bradley HS
Christina Bice - AP Calculus AB/BC	Davidson HS
William Kuch - AP Calculus AB/BC	Davidson HS
Noura Elnaji - Intervention Specialist	Davidson HS
Emma Longauer - Intervention Specialist	Davidson HS
John Klie - Intervention Specialist	Weaver MS
Max Mordarski - Intervention Specialist	Station
Elissa Shoaf - Intervention Specialist	Darby HS
Matt Anderson - Assistant Principal	Darby HS
Adam Olsheski - Assistant Principal	Darby HS
Erin Dooley - Principal	Station
Amy Case - Special Education Coordinator	Central Office
Jennifer Lowery - Gifted Coordinator	Central Office
Mark Tremayne - Director of Innovation and Extended Learning	Central Office
Molly Walker - Director of Measurement, Intervention and Enrichment	Central Office
Hilary Sloat - Director of Diversity, Equity and Inclusion	Central Office
Herb Higginbotham - Director of Elementary Education	Central Office
Jacob Grantier - Director of Secondary Education	Central Office
Cori Kindl - Executive Director of Curriculum and Instruction	Central Office

Introduction

As the world evolves, so too must teaching and learning. Today's student, at a glance, looks similar to the students of yesterday; however, on the inside, this student is really quite different. In general, 21st century students are walking into our classrooms with access to knowledge and content at their fingertips. They are more experienced and ready to engage in complex learning and real-world problem-solving. Additionally, today's global society, sparked by rapid technological advances and innovation is putting new demands on a global work-force. Students must possess a new set of skills and competencies to be successful in the future. As such, school districts must consistently and systematically review what is taught in school and how it is taught. The Hilliard City Schools Curriculum Department works alongside teachers, administrators, families, and the community to define and communicate what all students will know and be able to do at each grade level and within each course in order to be Ready for Tomorrow.

The Mathematics Course of Study is the district's foundational document which outlines the K-12 Curriculum Program for Mathematics. The Course of Study is designed, developed, and revised periodically to ensure that the most recent Ohio Learning Standards are taught with fidelity, incorporating current research within Mathematics and using evidence-based instructional strategies and practices to maximize students' knowledge and skills. In addition, resources are evaluated for alignment and intentionality. The Course of Study consists of several key components, including a foreword, table of contents, introduction, the district's philosophy and vision statement, the district's educational goals, the content area's vision and instructional commitments, the K-12 Ohio Mathematical Learning Standards, a scope and sequence for each grade level and/or course, and assessment practices.

When revising this course of study, the following areas of mathematics instruction were at the forefront of professional development to guide the design of this document:

- Mathematical Teaching Practices
- Ohio Mathematics Model Curriculum
- Conceptual Understanding and Procedural Fluency
- Engaging Students in Deep Mathematical Thinking
- Equitable Instructional Practices
- Supporting Productive Struggle
- Culturally Responsive Practices

The resources and research listed below anchored the work of revising the K-12 Mathematics curriculum:

- Building Thinking Classrooms in Mathematics: 14 Teaching Practices for Enhancing Learning (2021) - Building Thinking Classrooms is a framework for organizing a mathematics classroom involving 14 principles used to help teachers structure their math classes around student thinking.This framework, based on 15 years of classroom research, relies on students working in visibly random groups on vertical non-permanent surfaces (such as vertical whiteboards). This purposeful strategy, in collaboration with 13 more, provides the opportunity to utilize visible student and group thinking to build a community of thinkers.
- Catalyzing Change in Early Childhood and Elementary Mathematics (2020) - This resource recognizes that the strengths and needs of young children must be considered when addressing the continuity and alignment of mathematics education for this student group. It also identifies and addresses the critical conversations necessary to meet the following challenges:

broadening the purpose of school mathematics to prioritize development of deep conceptual understanding so that children experience joy and confidence in themselves as emerging mathematicians; dismantling structural obstacles that impede mathematics working for each student; implementing equitable instructional practices to cultivate students' positive mathematical identities; organizing mathematics along a common shared pathway grounded in the use of mathematical practices and processes to coherently develop a strong foundation of deep mathematical understanding for each and every child.

- Catalyzing Change in Middle School Mathematics: Initiating Critical Conversations (2020) -This resource recognizes that the needs of young adolescents are different from elementary and high school-age students and that policies, practices, and issues must consider the unique needs of this student group. Students undergo significant developmental changes from elementary school to middle school. These changes contribute to how they see and understand the world as well as how they see and understand their place in the world.
- Catalyzing Change in High School Mathematics: Initiating Critical Conversations (2020) - This resource identifies and addresses critical challenges in high school mathematics to ensure that each and every student has the mathematical experiences necessary for his or her future personal and professional success. Some of these challenges include a focus on college and career readiness, dismantling obstacles so that each and every student can have access to higher mathematics courses, and implementing equitable instructional practices, among others.
- Culturally Responsive Teaching and the Brain (Hammond, 2014) - Culturally Responsive Practice is a systematic approach to teaching that recognizes a student's unique culture can strengthen a connectedness to school and enhance learning. In a culturally responsive classroom, student's lived experiences, cultures, and linguistic capital are recognized and valued, high expectations for learning are supported, high-quality, rigorous instruction is provided, and students are stretched cognitively to grow as independent learners
- Ohio's Learning Standards for Mathematics
- Ohio's Mathematics Model Curricula
- Principles to Action: Ensuring Mathematical Success For All (2014) - This resource connects research with practice. Specific, research-based teaching practices that are essential for a high-quality mathematics education for each and every student are combined with core principles to build a successful mathematics program at all levels. It includes eight, research-based, essential mathematics teaching practices; conditions, structures, and policies necessary to support effective teaching practices; implementation strategies for the Common Core State Standards for Mathematics built from Principles and Standards for School Mathematics and designed to attain much higher levels of mathematics achievement for each and every student; unproductive and productive beliefs, obstacles, and key actions that must be understood, acknowledged, and addressed by all stakeholders; and strategies for teachers to engage students in mathematical thinking, reasoning, and sense making to significantly strengthen teaching and learning.
- Productive Math Struggle; A 6-Point Action Plan for Fostering Perseverance (2020) - This resource guides teachers through six specific actions including valuing, fostering, building, planning, supporting, and reflecting on struggle to create a game plan for overcoming obstacles by sharing actionable steps, strategies and tools for implementation, as well as instructional tasks for each grade level.

- National Council of Teachers of Mathematics (NCTM)
- Standards for Mathematical Practice - The Standards for Mathematical Practice describe the skills that mathematics educators should seek to develop in their students. The practices rest on important processes and proficiencies with longstanding importance in mathematics education. The mathematical practices for each grade, together with Ohio's Learning Standards for Mathematics, prescribe that students experience mathematics as a rigorous, coherent, useful and logical subject.

District Mission and Educational Philosophy

District Mission: Hilliard City Schools will ensure that every student is Ready For Tomorrow.

Educational Philosophy, Purpose and Beliefs:
The district mission will be accomplished by:

1. Academics – The foundational knowledge we require all our students to be skilled in. The traditional focus of schools and our elite teachers as they prepare our students.

2. Interests – Connecting learning to life. We align students' strengths to their path after high school. This is accomplished by providing opportunities for students to discover their own potential.

3. Mindset – Our passion for growth leads to an understanding that change and improvement are a part of life. We foster student self awareness to guide students to an understanding of their personal strengths and weaknesses.

The purpose of the Hilliard City School District is to enable students to become productive citizens in an ever-changing world. We believe it is the responsibility of the District to ensure that all students can learn and grow.

1. Students are the focus of all school activities.

2. To develop all students' potential, the Hilliard City School District will strive to provide a safe and caring environment.

3. The District will guide students in the pursuit of excellence in knowledge and skills and prepare them to become productive citizens in a democratic society.

4. The District will provide ongoing professional learning for all staff, ensuring adequate facilities, resources and instructional tools essential to continuous student improvement.

5. A student's value system begins with the family.

6. Partnerships between home, school and community are essential to student success.

All building and course of study philosophies reflect and extend the Board's philosophy.

(Policy - AD, ADA)

District Instructional Goals

The educational goals for the District address the meaning of a quality education. Each learner who has the potential and inner strength should strive toward the ideal implicit in each goal.

The goals are intertwined: no one goal stands in isolation from the rest. They will help to define performance objectives for learners, identify tasks to be performed by teachers in giving substance to those objectives and help to determine means for evaluating learners' progress toward the goals.

1. **Physical and Emotional Well-Being** - Education should contribute to the learner's physical and emotional well-being, especially to a sense of self-worth and to a capacity for influencing one's own destiny through personal growth. Students will also learn to work effectively and to cooperate with others in order to form positive, healthy relationships.
2. **Communication Skills** - Education should develop in each learner the basic skills needed for communication, perception, evaluation, and conceptualization of ideas. Among the most important skills are reading, writing, speaking, listening, computational skills, visual literacy and technology literacy.
3. **Effective Use of Knowledge** - Education should provide each learner access to human cultural heritage. It should stimulate intellectual curiosity and promote intellectual development. Students should strive to produce high quality products based on knowledge work.
4. **Capacity and Desire for Lifelong Learning -** Education should foster and stimulate in each learner the natural desire for lifelong learning and should develop the skills necessary to fulfill that desire.
5. **Citizenship in a Democratic Society** - Education should provide each learner with an understanding of how our society functions in theory and in practice. Education must also foster individual commitment to exercise the rights and responsibilities of citizenship including participation in the democratic process and service to society.
6. **Respect for the Community of Man** - Education should provide each learner with the knowledge and experience which contribute to an understanding of human similarities and differences, thereby advancing mutual respect for humanity and for the dignity of the individual.
7. **Occupational Competence** - Education should provide the learner with the skills, experience, attitudes and understanding for future careers. It is also important for the learner to develop a capacity to adapt to change by solving problems and thinking creatively.
8. **Understanding of the Environment** - Education should provide each learner with knowledge and understanding of the social, physical, and biological worlds, and the balance between humans and their environment, and should develop attitudes and behavior leading to intelligent use of the environment. Students will learn to conserve the natural world in which they live.
9. **Creative Interests and Talents** - Education should provide each learner with varied opportunities to nurture interests, to discover and to develop natural talents and to express values and feelings through various media. Students should develop an appreciation of the arts, leisure and everyday life.
10. **Individual Values and Attitudes** - Education should expand and advance the humane dimensions of all learners, especially by helping them to identify and cultivate their own moral and ethical values and attitudes.

(Policy - IA)

Mathematics Vision Statement and Instructional Commitments

Vision Statement:
Hilliard City Schools' Mathematics vision is for all students to acquire the knowledge and skills to become mathematicians who reason, think critically and are prepared to contribute to a global community.

Instructional Commitments:
In order to achieve our vision, Hilliard City Schools teachers of Mathematics are committed to each of the following:

1. Partnerships with Students - Teachers of Mathematics will build relationships with students, fostering classroom communities that promote trust, intellectual freedom, and innovation. By building in each student a positive mathematical identity and sense of agency, teachers will cultivate collaborative partnerships with students, empowering them to reach their full potential.

2. Culturally Responsive Practices - Teachers of Mathematics create a classroom of access and equity, recognizing that all individuals bring unique experiences, backgrounds, cultural perspectives, and language to the classroom community. Teachers will use this understanding to inform instructional practices and support students in mastering mathematical concepts and skills. Students will engage with mathematics in ways that allow them to see themselves, as well as the perspectives of others, reflected in their learning. Teachers ensure that all students have access to a challenging mathematics curriculum, taught by skilled and effective teachers who differentiate, accommodate, and remediate mathematical instruction as needed.

3. Appropriately Challenging, Rigorous Instruction - Teachers of Mathematics will recognize that each and every student is able to solve challenging mathematical tasks successfully. Challenging tasks are at the core of lessons focused on mathematical reasoning, problem solving, and sense making, and these tasks help motivate students to learn more. Teachers must challenge students to persevere in order to give them the experience of success in meeting high expectations.

   Teaching with high expectations means giving each and every student access to challenging tasks, curriculum and courses that make reasoning and problem solving the focus for each student. Teaching mathematics with high expectations for all students invites students to learn to identify assumptions, develop arguments, and make connections within mathematical topics and to other contexts and disciplines.

4. Research-Based Practices - Teachers of Mathematics will provide students with high-quality,research-based mathematical instruction to meet students' individual needs. The implementation of research-based practices will improve student learning outcomes and equip students with the mathematical knowledge and skills necessary to engage in mathematical reasoning, problem solving, and sense making experiences.

5. Authenticity and Real-World Connections - Teachers of Mathematics will connect mathematical understandings to real-world applications by providing students with opportunities to explore the mathematics in the world around them. Students will be encouraged to question, critique, and create solutions to problems and concerns related to their lives. Students will explore mathematical concepts in their areas of interest, as well as extend to experiences with new connections. Teachers will create mathematically powerful learning environments that provide space for wondering and asking questions, allow students to experience the joy of mathematical understanding, and foster an appreciation for the beauty of mathematics.

6. Learner's Mindset - Teachers of Mathematics are committed to and collaborate for the mathematical success of every student and for personal and collective growth toward effective teaching and learning of mathematics. As professionals, mathematics teachers recognize that their own learning is never finished and continually seeks to improve and enhance their mathematical knowledge for teaching, their knowledge of mathematical pedagogy, and their knowledge of students as learners of mathematics.

Current Research and Best Practices for Mathematics

As referenced in the introduction, the K-12 Mathematics Course of Study revision relied on current research and evidence-based practices that should be implemented in every classroom in order for all students to acquire the knowledge and skills to become mathematicians who think critically and contribute to a global community. The following is a summation of several key pieces, including excerpts taken from Principles to Actions - Ensuring Mathematical Success for All (2014), Catalyzing Change in Elementary, Middle, and High School Mathematics (2018), Building Thinking Classrooms in Mathematics Grades K-12 (2021), Productive Math Struggle - A 6-Point Action Plan for Fostering Perseverance (2020), and Ohio Department of Education Model Curricula for Mathematics (2018).
The learning of mathematics has been defined to include the development of five interrelated strands that, together, constitute mathematical proficiency. They are:

1. Conceptual Understanding
2. Procedural Fluency
3. Strategic Competence
4. Adaptive Reasoning
5. Productive Disposition/Productive Struggle

Conceptual understanding is the ability to formulate, represent, and solve mathematical problems and adaptive reasoning is the capacity to think logically and to justify one's thinking. Together, these two strands establish the foundation and are necessary for building procedural fluency, which is the meaningful and flexible use of procedures to solve problems.

Strategic competence is the ability to formulate, represent, and solve mathematical problems. Adaptive reasoning is the capacity to think logically and to justify one's thinking. These two strands reflect the need for students to develop mathematical ways of thinking as a basis for solving mathematical problems that they may encounter in real life, as well as within mathematics and other disciplines.

Productive disposition/Productive struggle is the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that consistent effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics. It is critical that all students recognize the value of studying mathematics and believe that they are capable of learning mathematics through grit and effort. It is critical that all students recognize the value of studying mathematics and believe that they are capable of learning mathematics through grit and effort.

These ways of thinking are represented in the 8 Mathematical Practices which represent what students are doing as they learn mathematics. Hilliard's K-12 Mathematics Course of Study is grounded in these 8 Mathematical Practices and believe that they should be developed in all students. These standards are explained in more depth later in this document but, in short, the 8 Mathematical Learning Practices are:

1. Make Sense of Problems and Persevere in Solving Them
2. Reason Abstractly and Quantitative
3. Construct Viable Arguments and Critique the Reasoning of Others
4. Model with Mathematics
5. Use Appropriate Tools Strategically

6. Attend to Precision
7. Look For and Make Use of Structure
8. Look For and Express Regularity in Repeated Reasoning

Mathematical Teaching Practices

Hilliard City School District understands that effective teaching of mathematics can be different than in other disciplines. We believe that mathematical learning is an active process, in which each student builds his or her mathematical understanding from personal experiences, coupled with timely and targeted feedback. Research from the National Council of Mathematics (NCTM) and others have identified a number of principles of learning that provide a foundation for effective mathematics teaching. Specifically, learners should have experiences that enable them to:

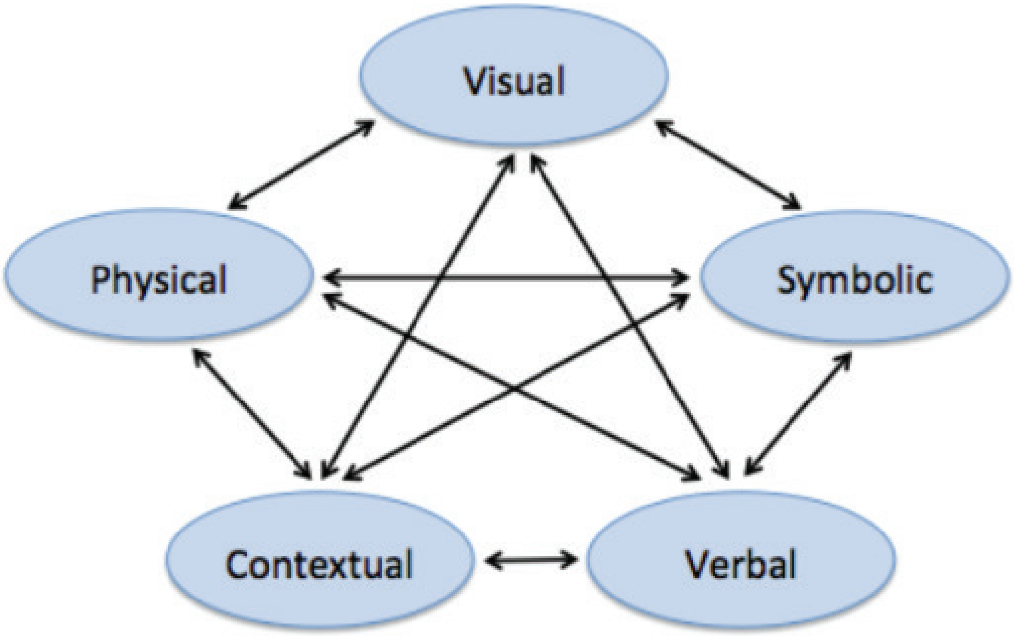
- Engage with challenging tasks that involve active meaning making and support meaningful learning;
- Connect new learning with prior knowledge and informal reasoning, and, in the process, address preconceptions and misconceptions;
- Acquire conceptual knowledge as well as procedural knowledge, so that they can meaningfully organize their knowledge, acquire new knowledge, and transfer and apply knowledge to new situations;
- Construct knowledge socially, through discourse, activity, and interaction related to meaningful problems;
- Receive descriptive and timely feedback so that they can reflect on and revise their work, thinking, and understanding; and
- Develop metacognitive awareness of themselves as learners, thinkers, and problem solvers, and learn to monitor their learning and performance.

In the book, Building Thinking Classrooms in Mathematics, each chapter explores one of the fourteen optimal practices, beginning with a deep dive into research regarding the institutionally normative practices that exist in many classrooms around the world. It reveals how each of these practices is working against our efforts to get students to think, and then it offers a clear presentation of what the research revealed to be the optimal practice for each variable, unpacking it into macro- and micro-practices. When all fourteen of these optimal practices are enacted together, a teacher will have a classroom that is not only conducive to thinking but also requires it. They will have a space that is inhabited by thinking individuals as well as individuals thinking collectively, learning together, and constructing knowledge and understanding through activity and discussion. They will have built a thinking classroom. (Building Thinking Classrooms in Mathematics, Liljdahl, 2020).

Use and Connect Mathematical Representations

According to the National Research Council (2001), "Because of the abstract nature of mathematics, people have access to mathematical ideas only through the representation of those ideas" (p. 94). Visual representations are of particular importance in the mathematics classroom.

These representations include visual, symbolic, verbal, contextual, and physical, each one providing a new dimension in which to view mathematical concepts. The diagram shown here illustrates conceptual networks developed when using various representations (NCTM, 2014, p. 25).



In addition, the Concrete-Representational-Abstract teaching approach is where students work with hands-on materials that represent mathematics problems (concrete), pictorial representations of mathematics problems (representational), and mathematics problems with numbers and symbols (abstract). The teacher explicitly bridges the connection between the concrete, representational, and abstract representations of the mathematics problems.

Committed to High Quality Instruction

The National Council of Teachers of Mathematics (NCTM) states that every student deserves an excellent program of instruction in mathematics that challenges each student to achieve at the high level required for productive citizenship and employment. Teachers should guide the learning process in their classrooms and manage the classroom environment through a variety of instructional approaches directly tied to the mathematics content and to students' needs. Learning mathematics is maximized when teachers focus on mathematical thinking and reasoning. They believe that learning mathematics is enhanced when content is placed in context and is connected to other subject areas and when students are given multiple opportunities to apply mathematics in meaningful ways as part of the learning process. Additionally, students should be encouraged and supported when using diverse strategies and different algorithms to solve problems, and teachers must recognize and take advantage of these alternative approaches to help students develop a better understanding of mathematics. (NCTM, 2018)

Hilliard City School District is committed to high-quality instruction and ensuring that each student is ready for rigor and independent learning in every classroom, every day. Through teaching the Mathematics Content Standards, we are committed to eliminate persistent racial, ethnic, and income achievement gaps so that all students have opportunities and support to achieve high levels of mathematics learning. Additionally, it is important that we increase the level of mathematics learning of all students, so that they are college and career ready when they graduate from high school, and finally, we need to increase the number of high school graduates, especially those from traditionally underrepresented groups, who are interested in, and prepared for, STEM careers. In short, we must move from "pockets of excellence" to "systemic excellence" by providing mathematics education that supports the learning of all students at the highest possible level. (Principles to Action, NCTM, 2014)

A Balanced Assessment System

A balanced assessment framework allows all learners to demonstrate their understanding, all teachers to use results as a means of providing responsive instruction and intervention, and all stakeholders to recognize areas of strength and need in support of every student, without exception.
Battelle for Kids, as part of their Assessment21 professional learning series, identified four big ideas regarding assessments and how they can be leveraged to drive deeper learning.

- Testing is an event. Assessment is a process.
- Assessing deeper learning cannot be done in a vacuum.
- Assessment for deeper learning promotes transfer.
- Students are important stakeholders in the assessment process - now more than ever.

A combination of diagnostic, formative, and summative assessments provide learners and educators with valuable information to ensure that the learning environment is responsive to the diverse needs of all students and provides equitable opportunities to engage with academics, interests, and mindsets in a culturally relevant way. Assessments for, as, and of learning allow teachers and students to gather, examine, and use data in support of deep learning and thinking.

Assessments for learning are intended to occur during the learning process to gather specific information about each student's learning path based on what they know and can do. These opportunities work to unlock prior knowledge, identify misconceptions and errors in thinking, and demonstrate understanding and progress toward mastery of a particular standard or outcome. These assessments should be designed such that teachers can easily unpack and use the information to differentiate instruction, provide targeted and responsive interventions, and create conditions so that they, in partnership with students, can identify successful next steps in the learning process. Assessments for learning also provide each student with accurate and descriptive feedback and help all stakeholders gain an understanding of achievement, progress, and any necessary support.

Assessments as learning serve as opportunities to promote self-assessment and self-monitoring. In order for students to adequately plan for learning, connect new ideas to existing understandings, monitor progress, identify misconceptions, make sense of new concepts, and reflect on learning, teachers must both support the ambiguity and uncertainty that is inevitable with new learning as well as model and guide mechanisms of questioning one's own thinking.

Assessments of learning serve as a summary of student achievement and often represent summative demonstrations of mastery. These assessments are meant to be fair and accurate sources of information regarding student progress toward identified outcomes and can be used, when appropriate, to make educational decisions about and for students. To ensure these assessments are reliable, valid, and accurate representations of student learning, they should be transparent, aligned to curricular goals and outcomes, and accurately reflect the rigor of the course and intended learning.

The Hilliard City School District strives to accurately measure student achievement using a balanced assessment system. A single data point has limitations and tells only a part of the full picture of the district and a student's academic performance. By utilizing multiple data points, we can create a robust picture of student achievement that allows us to truly prepare students to be Ready for Tomorrow. In creating that balanced assessment system, each assessment type has unique benefits. Listed below are some of the roles of the major assessments in our system.

1. **Purpose of Classroom Teacher Assessments:**
    • Monitor student progression on mastery of state standards
    •Identify common student misconceptions
    • Identify where to adjust instruction
    • Identify student strengths and weaknesses
    • Help inform student grades
    • Communication tool for students to benchmark their learning

2. **Purpose of Common District Assessments (in Performance Matters):**

    For Teachers:
    • Monitor student progression on mastery of state standards
    • Identify common student misconceptions
    • Identify where to adjust instruction
    • Identify student strengths and weaknesses
    • Compare student progress to other students in the building and district
    • Encourage collaboration in data analysis and instructional planning

    For Building Leaders:
    • Monitor student progression on mastery of state standards
    • Compare student progress to other students in the district
    • Identify where teachers need PD and/or support
    • Identity areas for celebration and improvement
    • Lead data team discussions and encourage collaboration
    • Monitor building progress toward state assessment goals
    • Identify trends among student groups

    For District Leaders:
    • Identify where buildings/teachers need support/PD
    • Identify district learning gaps
    • Inform district improvement planning
    • Monitor student progression toward master of state standards
    • Identify resources needed for support and justify the investment in those resources
    • Identify trends with subgroups or other identified populations
    • Monitor consistency in student achievement district wide

3. **Purpose for STAR (Renaissance) Assessments:**
    • Impartial, third party, look at student achievement
    • Calculates nationally normed, comparative, student growth data
    • Monitors student mastery of state standards and progress toward success on state assessments
    • Allows for student data comparison over time, including multiple years
    • Allows for the identification and monitoring of academic interventions
    • Allows for progress monitoring for EL students, students on IEPs, or other students as needed
    •Inform district improvement planning

4. **Purpose for Ohio State Assessments:**
    - Impartial, third party, look at student achievement
    - Identify district curriculum gaps
    - Identify where buildings/teachers need support/PD
    - Identify student achievement and mastery of state standards district wide
    - Creates comparative growth data
    - Inform district improvement planning
    - Identify trends with subgroups or other identified populations
    - Allows students to demonstrate competency toward graduation pathways
    - Are the basis for the state's school evaluation system (District Report Card)
    - Evaluation of our district progress in comparison to other district in the state
    - Evaluate student skills in preparation for post secondary options

Ohio K-12 Mathematics Standards Organization and Overview

The K-12 Mathematics Course of Study aligns to the Ohio Mathematical Practice and Grade Level/Course Learning Standards. It establishes a foundation for the planning and development of lessons, resource selection and instruction. The standards are research and evidence-based, aligned with college and work expectations, rigorous, and internationally benchmarked. They define what students should understand and be able to do in their study of mathematics. Asking a student to understand something means asking a teacher to assess whether the student has understood it. One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is true, or where a mathematical rule comes from. There is a world of difference between a student who can summon a mnemonic device to expand a product such as (a + b)(x + y) and a student who can explain where the mnemonic device comes from. The student who can explain the rule understands the mathematics at a much deeper level. Mathematical understanding and procedural skill are equally important, and both are accessible using mathematical tasks of sufficient richness.

Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently, and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving more complicated problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others. Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics. Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically. Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision. Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently and express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure. Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression x2 + 9x + 14, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complex things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see 5 – 3(x – y) 2 as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. Look for and express regularity in repeated reasoning. Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, students might abstract the equation (y - 2)/(x - 1) = 3. Noticing the regularity in the way terms cancel when expanding (x - 1)(x + 1), (x - 1)(x2 + x + 1), and (x - 1)(x3 + x2 + x + 1) might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years. The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices. In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

K-12 Mathematics
Graded Course of Study

Grade Level Standards
Scope and Sequence

HILLIARD CITY SCHOOLS

The following Graded Course of Study identifies the Math Standards taught at each grade level and/or course as well as provides a sample Scope and Sequence that illustrates how standards are paired together and sequenced throughout the course of a school year. The Scope and Sequence is a flexible guide and should be adaptive based on student learning. The Elementary Scope and Sequence, grades K-5, reflects all of the standards that could be potentially touched upon in a given trimester. The Secondary Scope and Sequence, grades 6-12, reflects the specific standards assessed in a unit. This is due to the fact that many Ohio Math Standards have multiple touch points throughout a given year, as well as within multiple courses. This symbol ★ denotes a standard that doubles as a Modeling Standard as well

Kindergarten Math Standards

Counting and Cardinality

Know number names and the count sequence.

K.CC.1	Count to 100 by ones and by tens.
K.CC.2	Count forward within 100 beginning from any given number other than 1.
K.CC.3	Write numerals from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).

Count to tell the number of objects.

K.CC.4	Understand the relationship between numbers and quantities; connect counting to cardinality using a variety of objects including pennies. a. When counting objects, establish a one-to-one relationship by saying the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. b. Understand that the last number name said tells the number of objects counted and that the number of objects is the same regardless of the arrangement or the order in which counted. c. Understand that each successive number name refers to a quantity that is one larger.
K.CC.5	Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1-20, count out that many objects.

Compare Numbers

K.CC.6	Orally identify (without using inequality symbols) whether the number of objects in one group is greater/more than, less/fewer than, or the same as the number of objects in another group, not to exceed 10 objects in each group.
K.CC.7	Compare (without using inequality symbols) two numbers between 0 and 10 when presented as written numerals.

Operations and Algebraic Thinking

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

K.OA.1	Represent addition and subtraction with objects, fingers, mental images, drawings, sounds such as claps, acting out situations, verbal explanations, expressions, or equations. Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)
K.OA.2	Solve addition and subtraction problems (written or oral), and add and subtract within 10 by using objects or drawings to represent the problem.

K.OA.3	Decompose numbers and record compositions for numbers less than or equal to 10 into pairs in more than one way by using objects and, when appropriate, drawings or equations.
K.OA.4	For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or, when appropriate, an equation.
K.OA.5	FluentlyG add and subtract within 5.

Numbers and Operations in Base Ten

Work with numbers 11–19 to gain foundations for place value.

K.NBT.1	Compose and decompose numbers from 11 to 19 into a group of ten ones and some further ones by using objects and, when appropriate, drawings or equations; understand that these numbers are composed of a group of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

Measurement and Data

Identify, describe, and compare measurable attributes.

K.MD.1	Identify and describe measurable attributes (length, weight, and height) of a single object using vocabulary terms such as long/short, heavy/light, or tall/short.
K.MD.2	Directly compare two objects with a measurable attribute in common to see which object has "more of" or "less of" the attribute, and describe the difference. For example, directly compare the heights of two children, and describe one child as taller/shorter.

Classify objects and count the number of objects in each category.

K.MD.3	Classify objects into given categories; count the numbers of objects in each category and sort the categories by count. The number of objects in each category should be less than or equal to ten. Counting and sorting coins should be limited to pennies.

Geometry

Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

K.G.1	Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind,and next to.
K.G.2	Correctly name shapes regardless of their orientations or overall size.
K.G.3	Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid").

Describe, compare, create, and compose shapes.

K.G.4	Describe and compare two- or three-dimensional shapes, in different sizes and orientations, using informal language to describe their commonalities, differences, parts, and other attributes.
K.G.5	Model shapes in the world by building shapes from components (such as sticks and clay balls) and drawing shapes.
K.G.6	Combine simple shapes to form larger shapes.

Kindergarten Scope and Sequence

Pre-work	Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7	Unit 8
Jo Boaler: First Days of School	Math in Our World	Numbers 1-10	Flat Shapes All Around Us	Understanding Addition and Subtraction	Composing and Decomposing Numbers to 10	Numbers 1-20	Solid Shapes All Around Us	Put It All Together
Community Building, Exploration, Math Mindset & Routines, Number Writing Strokes (at teacher discretion)	K.CC.1 K.CC.4	K.CC.1 K.CC.3 K.CC.4 K.CC.5 K.CC.6 K.CC.7	K.G.1 K.G.2 K.G.4 K.G.5 K.G.6 K.MD.2 K.MD.3	K.CC.1 K.CC.2 K.CC.3 K.CC.4 K.CC.5 K.OA.1 K.OA.2	K.CC.1 K.CC.2 K.CC.3 K.CC.5 K.OA.1 K.OA.2 K.OA.3 K.OA.4 K.OA.5	K.CC.1 K.CC.2 K.CC.3 K.CC.4 K.CC.5 K.OA.1 K.OA.2 K.OA.4 K.OA.5 K.NBT.1	K.G.1 K.G.2 K.G.3 K.G.4 K.G.5 K.G.6 K.MD.1 K.MD.2 K.MD.3	K.CC.1 K.CC.2 K.CC.3 K.CC.4 K.CC.5 K.CC.6 K.CC.7 K.MD.3 K.OA.1 K.OA.2 K.OA.3 K.OA.4 K.OA.5 K.NBT.1

First Grade Math Standards

Operations and Algebraic Thinking

Represent and solve problems involving addition and subtraction.

1.OA.1	Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. See Glossary, Table 1.
1.OA.2	Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)

Understand and apply properties of operations and the relationship between addition and subtraction.

1.OA.3	Apply properties of operations as strategies to add and subtract. For example, if 8 + 3 = 11 is known, then 3 + 8 = 11 is also known (Commutative Property of Addition); to add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12 *(Associative Property of Addition)*. Students need not use formal terms for these properties.
1.OA.4	Understand subtraction as an unknown-addend problem. For example, subtract 10 - 8 *by finding the number that makes 10 when added to 8.*

Add and subtract within 20.

1.OA.5	Relate counting to addition and subtraction, e.g., by counting onG 2 to add 2.
1.OA.6	Add and subtract within 20, demonstrating fluencyG with various strategies for addition and subtraction within 10. Strategies may include counting on; making ten, e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14; decomposing a number leading to a ten, e.g., 13 - 4 = 13 - 3 - 1 = 10 - 1 = 9; using the relationship between addition and subtraction, e.g., knowing that 8 + 4 = 12, one knows 12 - 8 = 4; and creating equivalent but easier or known sums, e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13.

Work with addition and subtraction equations.

1.OA.7	Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. *For example, which of the following equations are true and which are false?* 6 = 6; 7 = 8 – 1; 5 + 2 = 2 + 5; 4 + 1 = 5 + 2.
1.OA.8	Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations* 8 + ÿ = 11, 5 = ÿ – 3, 6 + 6 = ÿ.

Numbers and Operations in Base Ten

Extend the counting sequence.

1.NBT.1	Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.

Understand place value.

1.NBT.2	Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: 10 can be thought of as a bundle of ten ones - called a "ten;" the numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones; and the numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).
1.NBT.3	Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.

Use place value understanding and properties of operations to add and subtract.

1.NBT.4	Add within 100, including adding a two-digit number and a one-digit number and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; record the strategy with a written numerical method (drawings and, when appropriate, equations) and explain the reasoning used. Understand that when adding two-digit numbers, tens are added to tens; ones are added to ones; and sometimes it is necessary to compose a ten.
1.NBT.5	Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.
1.NBT.6	Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Measurement and Data

Measure lengths indirectly and by iterating length units.

1.MD.1	Order three objects by length; compare the lengths of two objects indirectly by using a third object.
1.MD.2	Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.

Work with time and money.

1.MD.3	Work with time and money. a. Tell and write time in hours and half-hours using analog and digital clocks. b. Identify pennies and dimes by name and value.

Represent and interpret data.

1.MD.4	Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.

Geometry

Reason with shapes and their attributes.

1.G.1	Distinguish between defining attributes, e.g., triangles are closed and three-sided, versus non-defining attributes, e.g., color, orientation, overall size; build and draw shapes that possess defining attributes.
1.G.2	Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape. Students do not need to learn formal names such as "right rectangular prism."
1.G.3	Partition circles and rectangles into two and four equal shares, describe the shares using the words halves, fourths, and quarters, and use the phrases half of, fourth of, and quarter of. Describe the whole as two of or four of the shares in real-world contexts. Understand for these examples that decomposing into more equal shares creates smaller shares.

First Grade Scope & Sequence

Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7	Unit 8
Adding, Subtracting, and Working with Data	**Addition and Subtraction Story Problems**	**Adding and Subtracting Within 20**	**Numbers to 99**	**Adding Within 20**	**Length Measurements Within 120 Units**	**Geometry and Time**	**Putting it All Together**
*Math Mindset & Routines 1.MD.C.4 1.OA.B.4 1.OA.C.5 1.OA.C.6	1.OA.A.1 1.OA.A.2 1.OA.B.3 1.OA.B.4 1.OA.C.5 1.OA.C.6 1.OA.D.7 1.OA.D.8 1.MD.C.4	1.OA.A.1 1.OA.A. 1.OA.B.3 1.OA.B.4 1.OA.A.C.5 1.OA.C.6 1.OA.D.7 1.OA.D.8 1.NBT.A.1 1.NBT.B.2	1.NBT.A.1 1.NBT.B.2 1.NBT.B.3 1.NBT.C.4 1.NBT.C.5 1.NBT.C.6 1.OA.A.1 1.OA.C.5 1.OA.C. 1.OA.D.7 1.OA.D.8	1.NBT.A.1 1.NBT.B.2 1.NBT.B.3 1.NBT.C.4 1.NBT.C.5 1.NBT.C.6 1.OA.A.1 1.OA.C.5 1.OA.C.6 1.OA.D.7 1.OA.D.8	1.MD.A.1 1.MD.A.2 1.NBT.A.1 1.NBT.B.3 1.NBT.C.4 1.NBT.C.5 1.OA.A.1 1.OA.A.2 1.OA.B.4 1.OA.C.5 1.OA.C.6 1.OA.C.7	1.G.A.1 1.G.A.2 1.G.A.3 1.MD.B.3 1.NBT.A.1 1.NBT.C.4 1.NBT.C.5 1.OA.C.6 1.OA.D.7	1.OA.A.1 1.OA.A.2 1.OA.C.6 1.OA.D.7 1.OA.D.8 1.NBT.A.1 1.NBT.B.3 1.NBT.C.4

Second Grade Math Standards

Operations and Algebraic Thinking

Represent and solve problems involving addition and subtraction.

2.OA.1	Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. See Glossary, Table 1.

Add and subtract within 20.

2.OA.2	FluentlyG add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers. See standard 1.OA.6 for a list of mental strategies.

Work with equal groups of objects to gain foundations for multiplication.

2.OA.3	Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.
2.OA.4	Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

Numbers and Operations in Base Ten

Understand place value.

2.NBT.1	Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases: a. 100 can be thought of as a bundle of ten tens - called a “hundred.” b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).
2.NBT.2	Count forward and backward within 1,000 by ones, tens, and hundreds starting at any number; skip-count by 5s starting at any multiple of 5.
2.NBT.3	Read and write numbers to 1,000 using base-ten numerals, number names, expanded formG, and equivalent representations, e.g., 716 is 700 + 10 + 6, or 6 + 700 + 10, or 6 ones and 71 tens, etc.
2.NBT.4	Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.

Use place value understanding and properties of operations to add and subtract.

2.NBT.5	Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.
2.NBT.6	Add up to four two-digit numbers using strategies based on place value and properties of operations.

2.NBT.7	Add and subtract within 1,000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; record the strategy with a written numerical method (drawings and, when appropriate, equations) and explain the reasoning used. Understand that in adding or subtracting three-digit numbers, hundreds are added or subtracted from hundreds, tens are added or subtracted from tens, ones are added or subtracted from ones; and sometimes it is necessary to compose or decompose tens or hundreds.
2.NBT.8	Mentally add 10 or 100 to a given number 100-900, and mentally subtract 10 or 100 from a given number 100-900.
2.NBT.9	Explain why addition and subtraction strategies work, using place value and the properties of operations. Explanations may be supported by drawings or objects.

Measurement and Data

Measure and estimate lengths in standard units.

2.MD.1	Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.
2.MD.2	Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.
2.MD.3	Estimate lengths using units of inches, feet, centimeters, and meters.
2.MD.4	Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.

Relate addition and subtraction to length.

2.MD.5	Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same whole number units, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)
2.MD.6	Represent whole numbers as lengths from 0 on a number line diagramG with equally spaced points corresponding to the numbers 0, 1, 2,..., and represent whole number sums and differences within 100 on a number line diagram.

Work with time and money.

2.MD.7	Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.
2.MD.8	Solve problems with money. a. Identify nickels and quarters by name and value. b. Find the value of a collection of quarters, dimes, nickels, and pennies. c. Solve word problems by adding and subtracting within 100, dollars with dollars and cents with cents (not using dollars and cents simultaneously) using the $ and symbols appropriately (not including decimal notation).

Represent and interpret data.

2.MD.9	Generate measurement data by measuring lengths of several objects to the nearest whole unit or by making repeated measurements of the same object. Show the measurements by creating a line plotG, where the horizontal scale is marked off in whole number units.
2.MD.10	Organize, represent, and interpret data with up to four categories; complete picture graphs when single-unit scales are provided; complete bar graphs when single-unit scales are provided; solve simple put-together, take-apart, and compare problems in a graph. See Glossary, Table 1.

Geometry

Reason with shapes and their attributes.

2.G.1	Recognize and identify triangles, quadrilaterals, pentagons, and hexagons based on the number of sides or vertices. Recognize and identify cubes, rectangular prisms, cones, and cylinders
2.G.2	Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.
2.G.3	Partition circles and rectangles into two, three, or four equal shares; describe the shares using the words halves, thirds, or fourths and quarters, and use the phrases half of, third of, or fourth of and quarter of. Describe the whole as two halves, three thirds, or four fourths in real-world contexts. Recognize that equal shares of identical wholes need not have the same shape.

Second Grade Scope and Sequence

Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7	Unit 8	Unit 9
Adding, Subtracting, and Working with Data	Adding and Subtracting within 100	Measuring Length	Addition and Subtraction on the Number Line	Numbers to 1,000	Geometry, Time, and Money	Adding and Subtracting within 1,000	Equal Groups	Putting It All Together
2.MD.10 2.OA.1 2.OA.2 2.NBT.2 2.NBT.5	2.MD.10 2.NBT.2 2.NBT.5 2.NBT.6 2.NBT.8 2.NBT.9 2.OA.1 2.OA.2	2.MD.1 2.MD.2 2.MD.3 2.MD.4 2.MD.5 2.MD.9 2.OA.1 2.OA.2 2.NBT.2 2.NBT.5	2.MD.5 2.MD.6 2.OA.1 2.NBT.2 2.NBT.5	2.NBT.1A 2.NBT.2 2.NBT.3 2.NBT.4 2.NBT.5 2.NBT.8 2.MD.6	2.G.1 2.G.3 2.MD.1 2.MD.7 2.MD.8C 2.NBT.1A 2.NBT.2 2.NBT.3 2.NBT.5 2.NBT.6 2.NBT.8 2.OA.1	2.NBT.1A 2.NBT.2 2.NBT.3 2.NBT.4 2.NBT.5 2.NBT.6 2.NBT.7 2.NBT.8 2.NBT.9 2.MD.10	2.NBT.2 2.NBT.7 2.NBT.8 2.OA.2 2.OA.3 2.OA.4 2.G.2	2.OA.1 2.OA.2 2.NBT.1A 2.NBT.3 2.NBT.5 2.NBT.7 2.NBT.9 2.MD.1 2.MD.4 2.MD.5 2.MD.9

Third Grade Math Standards

Operations and Algebraic Thinking

Represent and solve problems involving multiplication and division.

3.OA.1	Interpret products of whole numbers, e.g., interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each. (Note: These standards are written with the convention that a x b means a groups of b objects each; however, because of the commutative property, students may also interpret 5 x 7 as the total number of objects in 7 groups of 5 objects each).
3.OA.2	Interpret whole number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as* 56 ÷ 8.
3.OA.3	Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. See Glossary, Table 2. Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)
3.OA.4	Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 x ÿ = 48; 5 = ÿ ÷ 3; 6 x 6 = ÿ .

Understand properties of multiplication and the relationship between multiplication and division.

3.OA.5	Apply properties of operations as strategies to multiply and divide. For example, if 6 x 4 = 24 is known, then 4 x 6 = 24 is also known (Commutative Property of Multiplication); 3 x 5 x 2 can be found by 3 x 5 = 15, then 15 x 2 = 30, or by 5 x 2 = 10, then 3 x 10 = 30 (Associative Property of Multiplication); knowing that 8 x 5 = 40 and 8 x 2 = 16, one can find 8 x 7 as 8 x (5 + 2) = (8 x 5) + (8 x 2) = 40 + 16 = 56 (Distributive Property). Students need not use formal terms for these properties.
3.OA.6	Understand division as an unknown-factor problem. For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.

Multiply and divide within 100.

3.OA.7	FluentlyG multiply and divide within 100, using strategies such as the relationship between multiplication and division, e.g., knowing that 8 x 5 = 40, one knows 40 ÷ 5 = 8 or properties of operations. Limit to division without remainders. By the end of Grade 3, know from memory all products of two one-digit numbers.

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

3.OA.8	Solve two-step word problems using the four operations. Represent these problems using equations with a letter or a symbol, which stands for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. This standard is limited to problems posed with whole numbers and having whole number answers. Students may use parentheses for clarification since algebraic order of operations is not expected.

3.OA.9	Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.

Numbers and Operations in Base Ten

Use place value understanding and properties of operations to perform multi-digit arithmetic. A range of strategies and algorithms may be used.

3.NBT.1	Use place value understanding to round whole numbers to the nearest 10 or 100.
3.NBT.2	Fluently add and subtract within 1,000 using strategies and algorithmsG based on place value, properties of operations, and/or the relationship between addition and subtraction.
3.NBT.3	Multiply one-digit whole numbers by multiples of 10 in the range 10-90, e.g., 9 x 80, 5 x 60 using strategies based on place value and properties of operations.

Numbers and Operations– Fractions

Develop understanding of fractions as numbers. Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

3.NF.1	Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b.
3.NF.2	Understand a fraction as a number on the number line; represent fractions on a number line diagramG. a. Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line. b. Represent a fraction a/b (which may be greater than 1) on a number line diagram by marking off a lengths 1/b from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.
3.NF.3	Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. a. Understand two fractions as equivalent (equal) if they are the same size or the same point on a number line. b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model.G c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram. d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Measurement and Data

Solve problems involving money, measurement, and estimation of intervals of time, liquid volumes, and masses of objects.

3.MD.1	Work with time and money. a. Tell and write time to the nearest minute. Measure time intervals in minutes (within 90 minutes). Solve real-world problems involving addition and subtraction of time intervals (elapsed time) in minutes, e.g., by representing the problem on a number line diagram or clock. b. Solve word problems by adding and subtracting within 1,000, dollars with dollars and cents with cents (not using dollars and cents simultaneously) using the $ and symbol appropriately (not including decimal notation).
3.MD.2	Measure and estimate liquid volumes and masses of objects using standard units of grams, kilograms, and liters. Add, subtract, multiply, or divide whole numbers to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. Excludes multiplicative comparison problems involving notions of "times as much"; see Glossary, Table 2.

Represent and interpret data.

3.MD.3	Create scaled picture graphs to represent a data set with several categories. Create scaled bar graphs to represent a data set with several categories. Solve two-step “how many more” and “how many less” problems using information presented in the scaled graphs. *For example, create a bar graph in which each square in the bar graph might represent 5 pets, then determine how many more/less in two given categories.*
3.MD.4	Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by creating a line plotG, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

3.MD.5	Recognize area as an attribute of plane figures and understand concepts of area measurement. a. A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area. b. A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.
3.MD.6	Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).
3.MD.7	Relate area to the operations of multiplication and addition. a. Find the area of a rectangle with whole number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. b. Multiply side lengths to find areas of rectangles with whole number side lengths in the context of solving real-world and mathematical problems, and represent whole number products as rectangular areas in mathematical reasoning. c. Use tiling to show in a concrete case that the area of a rectangle with whole number side lengths a and b + c is the sum of a x b and a x c (represent the distributive property with visual models including an area model). d. Recognize area as additive. Find the area of figures composed of rectangles by decomposing into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real-world problems.

Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures

3.MD.8	Solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Geometry

Reason with shapes and their attributes.

3.G.1	Draw and describe triangles, quadrilaterals (rhombuses, rectangles, and squares), and polygons (up to 8 sides) based on the number of sides and the presence or absence of square corners (right angles).
3.G.2	Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.*

Third Grade Scope and Sequence

Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7	Unit 8
Introducing Multiplication	Area and Multiplication	Wrapping Up Addition and Subtraction within 1,000	Relating Multiplication to Division	Fractions as Numbers	Measuring Length, Time, Liquid Volume, and Weight	Two-Dimensional Shapes and Perimeter	Putting it All Together
3.MD.3 3.OA.1 3.OA.3 3.OA.4 3.OA.5 3.OA.9	3.OA.5 3.OA.1 3.MD.5 3.MD.7.d 3.NBT.2 3.MD.5.a 3.MD.5.b 3.MD.6 3.MD.7.a 3.MD.7.b	3.NBT.1 3.NBT.2 3.OA.5 3.OA.7 3.OA.8 3.OA.9 3.MD.1.b	3.NBT.2 3.OA.2 3.OA.3 3.MD.7.c 3.NBT.3 3.OA.6 3.OA.7 3.OA.7 3.OA.5 3.OA.8 3.MD.7 3.OA.4	3.G.2 3.NF.1 3.OA.7 3.NF.2 3.NF.2.a 3.NF.2.b 3.NF.3.c 3.NF.3.a 3.NF.3.b 3.OA.5	3.MD.4 3.NF.3.c 3.OA.7 3.MD.2 3.MD.1 3.MD.1.a 3.NBT.2 3.OA.3	3.G.1 3.NBT.3 3.OA.7 3.MD.8 3.NBT.2 3.OA.8	3.NF.1 3.NF.2 3.NF.2 3.MD.3 3.MD.7.b 3.MD.8 3.NBT.2 3.OA.8

Fourth Grade Math Standards

Operations and Algebraic Thinking

Use the four operations with whole numbers to solve problems.

4.OA.1	Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 x 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.
4.OA.2	Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. See Glossary, Table 2. Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)
4.OA.3	Solve multistep word problems posed with whole numbers and having whole number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Gain familiarity with factors and multiples.

4.OA.4	Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite.

Generate and analyze patterns.

4.OA.5	Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*

Numbers and Operations in Base Ten

Generalize place value understanding for multi-digit whole numbers less than or equal to 1,000,000.

4.NBT.1	Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right by applying concepts of place value, multiplication, or division.
4.NBT.2	Read and write multi-digit whole numbers using standard form, word form, and expanded formG. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.
4.NBT.3	Use place value understanding to round multi-digit whole numbers to any place through 1,000,000.

4.NBT.4	FluentlyG add and subtract multi-digit whole numbers using a standard algorithmG.
4.NBT.5	Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
4.NBT.6	Find whole number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Numbers and Operations – Fractions

Extend understanding of fraction equivalence and ordering limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

4.NF.1	Explain why a fraction a/b is equivalent to a fraction (n x a)/(n x b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
4.NF.2	Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100. (Fractions need not be simplified.)

4.NF.3	Understand a fraction *a/b* with a > 1 as a sum of fractions 1/b. a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction modelG. *Examples: 3/8 = 1/8 + 1/8 + 1/8; 3/8 = 1/8 + 2/8; 21/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.* c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.
4.NF.4	Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. a. Understand a fraction a/b as a multiple of 1/b. *For example, use a visual fraction model to represent 5/4 as the product 5 x (1/4), recording the conclusion by the equation 5/4 = 5 x (1/4), or 5/4 = 1/4 + 1/4 + 1/4 + 1/4 + 1/4.* b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 x (2/5) as 6 x (1/5), recognizing this product as 6/5. (In general, n x (a/b) = (n x a)/b.) c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

Understand decimal notation for fractions, and compare decimal fractions limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

4.NF.5	Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100. In general students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators, but addition and subtraction with unlike denominators is not a requirement at this grade.*
4.NF.6	Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.*
4.NF.7	Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.

Measurement and Data

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

4.MD.1	Know relative sizes of the metric measurement units within one system of units. Metric units include kilometer, meter, centimeter, and millimeter; kilogram and gram; and liter and milliliter. Express a larger measurement unit in terms of a smaller unit. Record measurement conversions in a two-column table. *For example, express the length of a 4-meter rope in centimeters. Because 1 meter is 100 times as long as a 1 centimeter, a two-column table of meters and centimeters includes the number pairs 1 and 100, 2 and 200, 3 and 300,...*
4.MD.2	Solve real-world problems involving money, time, and metric measurement. a. Using models, add and subtract money and express the answer in decimal notation. b. Using number line diagramsG, clocks, or other models, add and subtract intervals of time in hours and minutes. c. Add, subtract, and multiply whole numbers to solve metric measurement problems involving distances, liquid volumes, and masses of objects.
4.MD.3	Develop efficient strategies to determine the area and perimeter of rectangles in real-world situations and mathematical problems. *For example, given the total area and one side length of a rectangle, solve for the unknown factor, and given two adjacent side lengths of a rectangle, find the perimeter.*

Represent and interpret data.

4.MD.4	Display and interpret data in graphs (picture graphs, bar graphs, and line plotsG) to solve problems using numbers and operations for this grade.

Geometric measurement: understand concepts of angle and measure angles.

4.MD.5	Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement. a. Understand an angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a “one-degree angle,” and can be used to measure angles. b. Understand an angle that turns through n one-degree angles is said to have an angle measure of n degrees.

4.MD.6	Measure angles in whole number degrees using a protractor. Sketch angles of specified measure.
4.MD.7	Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

Geometry

Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

4.G.1	Draw points, lines, line segments, rays, angles (right, acute, and obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.
4.G.2	Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines or the presence or absence of angles of a specified size.

Fourth Grade Scope and Sequence

Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7	Unit 8	Unit 9
Factors and Multiples	Fraction Equivalence and Comparison	Extending Operations to Fractions	From Hundredths to Hundred-thousands	Multiplicative Comparison and Measurement	Multiplying and Dividing Multi-digit Numbers	Angles and Angle Measurement	Properties of Two-dimensional Shapes	Putting It All Together
4.OA.4 4.OA.5 4.OA.3	4.NF.2 4.NF.1 4.NF.4	4.NF.4.a 4.NF.4.c 4.NF.4.b 4.NF.3 4.NF.3.b 4.NF.3.a 4.NF.3.c 4.NF.3.d	4.NF.6 4.NF.7 4.NBT.1 4.NBT.2 4.NBT.4 4.NBT.3	4.OA.1 4.OA.2 4.NBT.5 4.MD.1 4.MD.2 4.MD.2.a 4.MD.2.b 4.MD.2.c 4.MD.3	4.NBT.5 4.NBT.6	4.G.1 4.MD.5 4.MD.5.a 4.MD.7 4.MD.5.b 4.MD.6	4.G.1 4.G.2 4.NF.4 4.NF.3.c 4.NBT.5 4.MD.3	4.OA.4 4.OA.5 4.OA.3

Fifth Grade Math Standards

Operations and Algebraic Thinking

Write and interpret numerical expressions.

5.OA.1	Use parentheses in numerical expressions, and evaluate expressions with this symbol. Formal use of algebraic order of operations is not necessary.
5.OA.2	Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation "add 8 and 7, then multiply by 2" as 2 x (8 + 7). Recognize that 3 x (18,932 + 921) is three times as large as 18,932 + 921, without having to calculate the indicated sum or product.

Analyze patterns and relationships.

5.OA.3	Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*

Numbers and Operations in Base Ten

Understand the place value system.

5.NBT.1	Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.
5.NBT.2	Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole number exponents to denote powers of 10.
5.NBT.3	Read, write, and compare decimals to thousandths. a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded formG, e.g., 347.392 = 3 x 100 + 4 x 10 + 7 x 1 + 3 x (1/10) + 9 x (1/100) + 2 x (1/1000). b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
5.NBT.4	Use place value understanding to round decimals to any place, millions through hundredths.

Perform operations with multi-digit whole numbers and with decimals to hundredths.

5.NBT.5	FluentlyG multiply multi-digit whole numbers using a standard algorithmG.
5.NBT.6	Find whole number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
5.NBT.7	Solve real-world problems by adding, subtracting, multiplying, and dividing decimals using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction, or multiplication and division; relate the strategy to a written method and explain the reasoning used. a. Add and subtract decimals, including decimals with whole numbers, (whole numbers through the hundreds place and decimals through the hundredths place). b. Multiply whole numbers by decimals (whole numbers through the hundreds place and decimals through the hundredths place). c. Divide whole numbers by decimals and decimals by whole numbers (whole numbers through the tens place and decimals less than one through the hundredths place using numbers whose division can be readily modeled). *For example, 0.75 divided by 5; 18 divided by 0.6; or 0.9 divided by 3.*

Numbers and Operations– Fractions

Use equivalent fractions as a strategy to add and subtract fractions. (Fractions need not be simplified.)

5.NF.1	Add and subtract fractions with unlike denominators (including mixed numbers and fractions greater than 1) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, use visual modelsG and properties of operations to show 2/3 + 5/4 = 8/12 + 15/12 = 23/12. In general, a/b + c/d = (a/b x d/d) + (c/d x b/b) = (ad + bc)/bd.*
5.NF2	Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction modelsG or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*

Apply and extend previous understandings of multiplication and division to multiply and divide fractions. (Fractions need not be simplified.)

5.NF.3	Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*

5.NF.4	Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. a. Interpret the product (a/b) x q as a parts of a partition of q into b equal parts, equivalently, as the result of a sequence of operations a x q ÷ b. *For example, use a visual fraction model to show (2/3) x 4 = 8/3, and create a story context for this equation. Do the same with (2/3) x (4/5) = 8/15. (In general, (a/b) x (c/d) = ac/bd.)* b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.
5.NF.5	Interpret multiplication as scaling (resizing). a. Compare the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. b. Explain why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence a/b = (nxa)/(nxb) to the effect of multiplying a/b by 1.
5.NF.6	Solve real-world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.
5.NF.7	Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. In general, students able to multiply fractions can develop strategies to divide fractions, by reasoning about the relationship between multiplication and division, but division of a fraction by a fraction is not a requirement at this grade. a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) x 4 = 1/3.* b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 x (1/5) = 4.* c. Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 pound of chocolate equally? How many 1/3 cup servings are in 2 cups of raisins?*

Measurement and Data

Convert like measurement units within a given measurement system.

5.MD.1	Know relative sizes of these U.S. customary measurement units: pounds, ounces, miles, yards, feet, inches, gallons, quarts, pints, cups, fluid ounces, hours, minutes, and seconds. Convert between pounds and ounces; miles and feet; yards, feet, and inches; gallons, quarts, pints, cups, and fluid ounces; hours, minutes, and seconds in solving multi-step, real-world problems.

Represent and interpret data.

5.MD.2	Display and interpret data in graphs (picture graphs, bar graphs, and line plotsG) to solve problems using numbers and operations for this grade, e.g., including U.S. customary units in fractions 1/2, 1/4, 1/8, or decimals.

Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

5.MD.3	Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. b. A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.
5.MD.4	Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.
5.MD.5	Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume. a. Find the volume of a right rectangular prism with whole number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole number products as volumes, e.g., to represent the Associative Property of Multiplication. b. Apply the formulas $V = \ell \times w \times h$ and $V = B \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole number edge lengths in the context of solving real-world and mathematical problems. c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real-world problems..

Geometry

Graph points on the coordinate plane to solve real-world and mathematical problems.

5.G.1	Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond, e.g., x-axis and x-coordinate, y-axis and y-coordinate.
5.G.2	Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

Classify two-dimensional figures into categories based on their properties.

5.G.3	Identify and describe commonalities and differences between types of triangles based on angle measures (equiangular, right, acute, and obtuse triangles) and side lengths (isosceles, equilateral, and scalene triangles).
5.G.4	Identify and describe commonalities and differences between types of quadrilaterals based on angle measures, side lengths, and the presence or absence of parallel and perpendicular lines, e.g., squares, rectangles, parallelograms, trapezoidsG, and rhombuses.

Fifth Grade Scope and Sequence

Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7	Unit 8
Finding Volume	Fractions as Quotients and Fraction Multiplication	Multiplying and Dividing Fractions	Wrapping Up Multiplication and Division with Multi-Digit Numbers	Place Value Patterns and Decimal Operations	More Decimal and Fraction Operations	Shapes on the Coordinate Plane	Putting it All Together
5.MD.3 5.MD.4 5.MD.5 5.OA.1 5. OA.2	5.NF.3 5.NF.4 5.OA.1	5.NF.4 5.NF.6 5.NF.7	5.MD.3 5.MD.5 5.NBT.5 5.NBT.6 5.OA.2 5.NF.3 5.NF.4	5.NBT.1 5.NBT.3 5.NBT.4 5.NBT.7 5.OA.1 5.OA.2 5.NF.4	5.MD.1 5.MD.2 5.NBT.1 5.NF.1 5.NF.2 5.NF.4 5.NF.5 5.OA.1	5.G.1 5.G.2 5.G.3 5.G.4 5.NBT.7 5.OA.2 5.OA.3	5.G.3 5.G.4 5.NBT.5 5.NBT.6 5.NBT.7 5.MD.3 5.MD.5 5.NF.1 5.NF.3 5.NF.4

Sixth Grade Math Standards
(Courses: Math 6, Honors Math 6)

Ratios and Proportional Relationships

Understand ratio concepts and use ratio reasoning to solve problems.

6.RP.1	Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*
6.RP.2	Understand the concept of a unit rate a/b associated with a ratio a:b with b 3 0, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger.*
6.RP.3	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagramsG, double number line diagramsG, or equations. a. Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?* c. Find a percent of a quantity as a rate per 100, e.g., 30% of a quantity means 30/100 times the quantity; solve problems involving finding the whole, given a part and the percent. d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

The Number System

Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

6.NS.1	Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction modelsG and equations to represent the problem. For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 pound of chocolate equally? How many 3/4 cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?

Compute fluently with multi-digit numbers and find common factors and multiples.

6.NS.2	Fluently divide multi-digit numbers using a standard algorithm.
6.NS.3	Fluently add, subtract, multiply, and divide multi-digit decimals using a standard algorithm for each operation.
6.NS.4	Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).*

Apply and extend previous understandings of numbers to the system of rational numbers.

6.NS.5	Understand that positive and negative numbers are used together to describe quantities having opposite directions or values, e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge; use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
6.NS.6	Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., - (-3) = 3, and that 0 is its own opposite. b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.
6.NS.7	Understand ordering and absolute value of rational numbers. a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. For example, interpret -3 > -7 as a statement that -3 is located to the right of -7 on a number line oriented from left to right. b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. For example, write -3°C > -7°C to express the fact that -3°C is warmer than -7 °C. c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. For example, for an account balance of -30 dollars, write |-30| = 30 to describe the size of the debt in dollars. d. Distinguish comparisons of absolute value from statements about order. For example, recognize that an account balance less than -30 dollars represents a debt greater than 30 dollars.
6.NS.8	Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Expressions and Equations

Apply and extend previous understandings of arithmetic to algebraic expressions.

6.EE.1	Write and evaluate numerical expressions involving whole number exponents
6.EE.2	Write, read, and evaluate expressions in which letters stand for numbers. a. Write expressions that record operations with numbers and with letters standing for numbers. For example, express the calculation "Subtract y from 5" as 5 – y. b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2(8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.* c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, using the algebraic order of operations when there are no parentheses to specify a particular order. *For example, use the formulas $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of length s = 1/2.*

6.EE.3	Apply the properties of operations to generate equivalent expressions. For example, apply the distributive property to the expression $3(2 + x)$ to produce the equivalent expression $6 + 3x$; apply the distributive property to the expression $24x + 18y$ to produce the equivalent expression $6(4x + 3y)$; apply properties of operations to $y + y + y$ to produce the equivalent expression $3y$.
6.EE.4	Identify when two expressions are equivalent, i.e., when the two expressions name the same number regardless of which value is substituted into them. For example, the expressions $y + y + y$ and $3y$ are equivalent because they name the same number regardless of which number y stands for.

Reason about and solve one-variable equations and inequalities.

6.EE.5	Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
6.EE.6	Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
6.EE.7	Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q, and x are all nonnegative rational numbers.
6.EE.8	Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

Represent and analyze quantitative relationships between dependent and independent variables.

6.EE.9	Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.

Geometry

Solve real-world and mathematical problems involving area, surface area, and volume.

6.G.1	Through composition into rectangles or decomposition into triangles, find the area of right triangles, other triangles, special quadrilaterals, and polygons; apply these techniques in the context of solving real-world and mathematical problems.
6.G.2	Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = \ell x w x h$ and $V = B x h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

6.G.3	Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.
6.G.4	Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Statistics and Probability

Develop understanding of statistical problem solving.

6.SP.1	Develop statistical reasoning by using the GAISE model: a. Formulate Questions: Recognize and formulate a statistical question as one that anticipates variability and can be answered with quantitative data. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because of the variability in students' ages. (GAISE Model, step 1)* b. Collect Data: Design and use a plan to collect appropriate data to answer a statistical question. (GAISE Model, step 2) c. Analyze Data: Select appropriate graphical methods and numerical measures to analyze data by displaying variability within a group, comparing individual to individual, and comparing individual to group. (GAISE Model, step 3) d. Interpret Results: Draw logical conclusions from the data based on the original question. (GAISE Model, step 4)
6.SP.2	Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.
6.SP.3	Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

Summarize and describe distributions.

6.SP.4	Display numerical data in plots on a number line, including dot plotsG (line plots), histograms, and box plotsG. (GAISE Model, step 3)
6.SP.5	Summarize numerical data sets in relation to their context. a. Report the number of observations. b. Describe the nature of the attribute under investigation, including how it was measured and its units of measurement. c. Find the quantitative measures of center (median and/or mean) for a numerical data set and recognize that this value summarizes the data set with a single number. Interpret mean as an equal or fair share. Find measures of variability (range and interquartile rangeG) as well as informally describe the shape and the presence of clusters, gaps, peaks, and outliers in a distribution. d. Choose the measures of center and variability, based on the shape of the data distribution and the context in which the data were gathered.

Math 6 & Honors Math 6* Scope & Sequence

(* Denotes additional standards for Honors Math 6)

	Ratio & Proportional Reasoning	The Number System	Expressions & Equations	Geometry	Statistics & Probability
SEMESTER 1					
Unit 1 • Operations with Fractions, Decimals & Whole Numbers		6.NS.1 6.NS.2 6.NS.3			
Unit 2 • Ratio, Rates, & Percents	6.RP.1 6.RP.2 6.RP.3a, b, c, d				
SEMESTER 2					
Unit 3 • Integers & The Coordinate Plane		6.NS.5 6.NS.6a,b,c 6.NS.7a,b 6.NS.8 7.NS.1a, b, c, d*		6.G.1 6.G.3 7.G.3*	
Unit 4 • Expressions			6.EE.1 6.EE.2a, b, c 6.EE.3 6.EE.4 6.EE.6		
Unit 5 • Equations & Inequalities			6.EE.5 6.EE.6 6.EE.7 6.EE.8 6.EE.9 7.EE.4a*		
Unit 6 • Area, Surface Area, & Volume			6.EE.7	6.G.1 6.G.2 6.G.4	
Unit 7 • Statistics					6.SP.1a, b, c 6.SP.2 6.SP.3 6.SP.4 6.SP.5a, b, c, d

Seventh Grade Math Standards
(Courses: Math 7, Accelerated Math 7)

Ratios and Proportional Relationships

Analyze proportional relationships and use them to solve real-world and mathematical problems.

7.RP.1	Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fractionG (1/2) /(1/4) miles per hour, equivalently 2 miles per hour.*
7.RP.2	Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.* d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.
7.RP.3	Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

The Number System

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

7.RP.1	Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. a. Describe situations in which opposite quantities combine to make 0. For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged. b. Understand p + q as the number located a distance |q| from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. c. Understand subtraction of rational numbers as adding the additive inverse, p - q = p + (-q). Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. d. Apply properties of operations as strategies to add and subtract rational numbers.
7.RP.2	Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (-1)(-1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

	b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then -(p/q) = (-p)/q = p/(-q). Interpret quotients of rational numbers by describing real-world contexts. c. Apply properties of operations as strategies to multiply and divide rational numbers. d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.
7.RP.3	Solve real-world and mathematical problems involving the four operations with rational numbers. Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

Expressions and Equations

Use properties of operations to generate equivalent expressions.

7.EE.1	Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.
7.EE.2	In a problem context, understand that rewriting an expression in an equivalent form can reveal and explain properties of the quantities represented by the expression and can reveal how those quantities are related. For example, a discount of 15% (represented by p - 0.15p) is equivalent to (1 - 0.15)p, which is equivalent to 0.85p or finding 85% of the original price.

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

7.EE.3	Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 93/4 inches long in the center of a door that is 271/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*
7.EE.4	Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form px + q = r and p(x + q) = r, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?* b. Solve word problems leading to inequalities of the form px +q > r or px + q < r, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example, as a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*

Geometry

Draw, construct, and describe geometrical figures and describe the relationships between them.

7.G.1	Solve problems involving similar figures with right triangles, other triangles, and special quadrilaterals. a. Compute actual lengths and areas from a scale drawing and reproduce a scale drawing at a different scale. b. Represent proportional relationships within and between similar figures.
7.G.2	Draw (freehand, with ruler and protractor, and with technology) geometric figures with given conditions. a. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. b. Focus on constructing quadrilaterals with given conditions noticing types and properties of resulting quadrilaterals and whether it is possible to construct different quadrilaterals using the same conditions.
7.G.3	Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

Solve real-life and mathematical problems involving angle measure, circles, area, surface area, and volume.

7.G.4	Work with circles. a. Explore and understand the relationships among the circumference, diameter, area, and radius of a circle. b. Know and use the formulas for the area and circumference of a circle and use them to solve real-world and mathematical problems.
7.G.5	Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.
7.G.6	Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Statistics and Probability

Use sampling to draw conclusions about a population.

7.SP.1	Understand that statistics can be used to gain information about a population by examining a sample of the population. a. Differentiate between a sample and a population. b. Understand that conclusions and generalizations about a population are valid only if the sample is representative of that population. Develop an informal understanding of bias.

Broaden understanding of statistical problem solving.

7.SP.2	Broaden statistical reasoning by using the GAISE model. a. Formulate Questions: Recognize and formulate a statistical question as one that anticipates variability and can be answered with quantitative data. *For example, "How do the heights of seventh graders compare to the heights of eighth graders?" (GAISE Model, step 1)* b. Collect Data: Design and use a plan to collect appropriate data to answer a statistical question. (GAISE Model, step 2)

	c. Analyze Data: Select appropriate graphical methods and numerical measures to analyze data by displaying variability within a group, comparing individual to individual, and comparing individual to group. (GAISE Model, step 3) d. Interpret Results: Draw logical conclusions and make generalizations from the data based on the original question. (GAISE Model, step 4)

Summarize and describe distributions representing one population and draw informal comparisons between two populations.

7.SP.3	Describe and analyze distributions. a. Summarize quantitative data sets in relation to their context by using mean absolute deviationG (MAD), interpreting mean as a balance point. b. Informally assess the degree of visual overlap of two numerical data distributions with roughly equal variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plotG (line plot), the separation between the two distributions of heights is noticeable.*

Investigate chance processes and develop, use, and evaluate probability models.

7.SP.5	Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event; a probability around 1/2 indicates an event that is neither unlikely nor likely; and a probability near 1 indicates a likely event.
7.SP.6	Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*
7.SP.7	Develop a probability modelG and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. a. Develop a uniform probability modelG by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.* b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*
7.SP.8	Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample spaceG for which the compound event occurs. b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language, e.g., "rolling double sixes," identify the outcomes in the sample space which compose the event. c. Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*

Math 7 & Accelerated Math 7* Scope & Sequence

(* Denotes additional standards for Accelerated Math 7)

	Ratio & Proportional Reasoning	The Number System	Expressions & Equations	Geometry	Statistics & Probability	Functions*
SEMESTER 1						
Unit 1 • Operations with Integers		7.NS.1a, b, c, d 7.NS.2a, b, c 7.NS.3	7.EE.2 7.EE.3			
Unit 2 • Operations with Rational Numbers		7.NS.1b, c 7.NS.2d 7.NS.3	7.EE.3			
Unit 3 • Expressions, Equations, & Inequalities		7.NS.1d 7.NS.2c	7.EE.1 7.EE.2 7.EE.3 7.EE.4a, b 8.EE.7a, b*			
Unit 4 • Ratios & Proportional Reasoning	7.RP.1 7.RP.2a,b,c,d 7.RP.3		7.EE.3			
SEMESTER 2						
Unit 5 • Percents	7.RP.3		7.EE.2 7.EE.3			
Unit 6 • Relationships in 2D Geometry			7.EE.3 7.EE.4b	7.G.1a, b 7.G.2a, b 7.G.5 8.G.5*		
Unit 7 • Measurement in 2D Geometry	8.NS.1-2		7.EE.4a 8.EE.2*	7.G.4a, b 7.G.6		
Unit 8 • Measurement in 3D Geometry			7.EE.4a 8.EE.2*	7.G.3 7.G.6 8.G.9*		
Unit 9 • Probability	7.RP.3				7.SP.5 7.SP.6 7.SP.7a, b 7.SP.8a, b, c	

Unit 10 • Statistics					7.SP.1a, b 7.SP.2a, b, c, d 7.SP.3a, b	
Unit 11 (Accelerated Math 7 Only) • Transformation & Congruence				8.G.1* 8.G.2* 8.G.3*		
Unit 12 (Accelerated Math 7 Only) • Similarity & Dilations				8.G.3* 8.G.4* 8.G.5*		
Unit 13 (Accelerated Math 7 Only) • Linear Relationships			8.EE.5* 8.EE.6*			8.F.2* 8.F.4*
Unit 14 (Accelerated Math 7 Only) Exponents & Scientific Notation			8.EE.1* 8.EE.3* 8.EE.4*			

Eighth Grade Math Standards
(Courses: Accelerated Math 7, Math 8, Algebra 1)

The Number System

Know that there are numbers that are not rational, and approximate them by rational numbers.

8.NS.1	Know that real numbers are either rational or irrational. Understand informally that every number has a decimal expansion which is repeating, terminating, or is non-repeating and non-terminating.
8.NS.2	Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions, e.g., π^2. *For example, by truncating the decimal expansion of $\sqrt{2}$, , show that $\sqrt{2}$, is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*

Expressions and Equations

Work with radicals and integer exponents.

8.EE.1	Understand, explain, and apply the properties of integer exponents to generate equivalent numerical expressions. *For example, 3^2 x 3-5 = 3-3 = 1/3^3 = 1/27.*
8.EE.2	Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.
8.EE.3	Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities and to express how many times as much one is than the other. *For example, estimate the population of the United States as 3 x 108; and the population of the world as 7 x 109; and determine that the world population is more than 20 times larger.*
8.EE.4	Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities, e.g., use millimeters per year for seafloor spreading. Interpret scientific notation that has been generated by technology.

Understand the connections between proportional relationships, lines, and linear equations.

8.EE.5	Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*
8.EE.6	Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b.

Analyze and solve linear equations and pairs of simultaneous linear equations.

8.EE.7	Solve linear equations in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form x = a, a = a, or a = b results (where a and b are different numbers). b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.
8.EE.8	Analyze and solve pairs of simultaneous linear equations graphically. a. Understand that the solution to a pair of linear equations in two variables corresponds to the point(s) of intersection of their graphs, because the point(s) of intersection satisfy both equations simultaneously. b. Use graphs to find or estimate the solution to a pair of two simultaneous linear equations in two variables. Equations should include all three solution types: one solution, no solution, and infinitely many solutions. Solve simple cases by inspection. *For example, 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6.* c. Solve real-world and mathematical problems leading to pairs of linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair. (Limit solutions to those that can be addressed by graphing.)*

Functions

Define, evaluate, and compare functions.

8.F.1	Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. Function notation is not required in Grade 8.
8.F.2	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*
8.F.3	Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function* $A = s^2$ *giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*

Use functions to model relationships between quantities.

8.F.4	Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
8.F.5	Describe qualitatively the functional relationship between two quantities by analyzing a graph, e.g., where the function is increasing or decreasing, linear or nonlinear. Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Geometry

Understand congruence and similarity using physical models, transparencies, or geometry software.

8.G.1	Verify experimentally the properties of rotations, reflections, and translations (include examples both with and without coordinates). a. Lines are taken to lines, and line segments are taken to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines.
8.G.2	Understand that a two-dimensional figure is congruentG to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. (Include examples both with and without coordinates.)
8.G.3	Describe the effect of dilationsG, translations, rotations, and reflections on two-dimensional figures using coordinates.
8.G.4	Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. (Include examples both with and without coordinates.)
8.G.5	Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

Understand and apply the Pythagorean Theorem.

8.G.6	Analyze and justify an informal proof of the Pythagorean Theorem and its converse.
8.G.7	Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
8.G.8	Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

8.G.9	Solve real-world and mathematical problems involving volumes of cones, cylinders, and spheres

Statistics and Probability

Investigate patterns of association in bivariate data.

8.SP.1	Construct and interpret scatter plots for bivariateG measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering; outliers; positive, negative, or no association; and linear association and nonlinear association. (GAISE Model, steps 3 and 4)

8.SP.2	Understand that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line. (GAISE Model, steps 3 and 4)
8.SP.3	Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height. (GAISE Model, steps 3 and 4)*
8.SP.4	Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

Math 8 Scope & Sequence

	The Number System	Expressions & Equations	Functions	Geometry	Statistics & Probability
SEMESTER 1					
Unit 1 • Linear Equations		8.EE.7a,b			
Unit 2 • Functions & Linear Relationships		8.EE.5 8.EE.6 8.EE.8a,b,c	8.F.1 8.F.2 8.F.3 8.F.4 8.F.5		
Unit 3 • Transformations & Congruence				8.G.1a, b, c 8.G.2 8.G.3	
Unit 4 • Special Angle Relationships				8.G.1b 8.G.5	
SEMESTER 2					
Unit 5 • Similarity & Dilations				8.G.3 8.G.4 8.G.5	
Unit 6 • Exponents & Scientific Notation		8.EE.1 8.EE.3 8.EE.4			
Unit 7 • Irrational Numbers & Pythagorean Theorem	8.NS.1 8.NS.2	8.EE.2		8.G.6 8.G.7 8.G.8	
Unit 8 • Volume of Curved Surfaces		8.EE.2		8.G.7 8.G.9	
Unit 9 • Exploring Data			8.F.4		8.SP.1 8.SP.2 8.SP.3 8.SP.4

Algebra 1 Standards
(Courses: Algebra 1, Mathematical Modeling & Reasoning, Data Science Foundations, Algebra 2, Algebra 3, PreCalculus, Statistics)

Number & Quantity

Reason quantitatively and use units to solve problems.

N.Q.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. ★
N.Q.2	Define appropriate quantities for the purpose of descriptive modeling. ★
N.Q.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. ★

Seeing Structure in Expressions

Interpret the structure of expressions

A.SSE.1	Interpret expressions that represent a quantity in terms of its context. ★ a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity.
A.SSE.2	Use the structure of an expression to identify ways to rewrite it. For example, to factor $3x(x - 5) + 2(x - 5)$, students should recognize that the "x - 5" is common to both expressions being added, so it simplifies to $(3x + 2)(x - 5)$; or see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

Write expressions in equivalent forms to solve problems.

A.SSE.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. ★ a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. c. Use the properties of exponents to transform expressions for exponential functions. For example, 8^t can be written as 2^{3t}

Arithmetic with Polynomials and Rational Expressions

Perform arithmetic operations on polynomials.

A.APR.1	Understand that polynomials form a system analogous to the integers, namely, that they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. a. Focus on polynomial expressions that simplify to forms that are linear or quadratic. (A1, M2)

Creating Equations

Create equations that describe numbers or relationships.

A.CED.1	Create equations and inequalities in one variable and use them to solve problems. Include equations and inequalities arising from linear, quadratic, simple rational, and exponential functions. ★ a. Focus on applying linear and simple exponential expressions. (A1, M1) b. Focus on applying simple quadratic expressions. (A1, M2)
A.CED.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. ★ a. Focus on applying linear and simple exponential expressions. (A1, M1) b. Focus on applying simple quadratic expressions. (A1, M2)
A.CED.3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods. ★ (A1, M1)
A.CED.4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. ★ a. Focus on formulas in which the variable of interest is linear or square. For example, rearrange Ohm's law V = IR to highlight resistance R, or rearrange the formula for the area of a circle A = (π)r2 to highlight radius r. (A1)

Reasoning with Equations and Inequalities

Understand solving equations as a process of reasoning and explain the reasoning.

A.REI.1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

Solve equations and inequalities in one variable.

A.REI.3	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
A.REI.4	Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. b. Solve quadratic equations as appropriate to the initial form of the equation by inspection, e.g., for $x^2 = 49$; taking square roots; completing the square; applying the quadratic formula; or utilizing the Zero-Product Property after factoring.

Solve systems of equations.

A.REI.5	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

A.REI.6	Solve systems of linear equations algebraically and graphically. a. Limit to pairs of linear equations in two variables. (A1, M1)
A.REI.7	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.

Represent and solve equations and inequalities graphically.

A.REI.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
A.REI.11	Explain why the x-coordinates of the points where the graphs of the equation $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, making tables of values, or finding successive approximations.
A.REI.12	Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Interpreting Functions

Understand the concept of a function, and use function notation.

F.IF.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
F.IF.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context
F.IF.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n + 1) = f(n) + f(n - 1)$ for n U 1.

Interpret functions that arise in applications in terms of the context.

F.IF.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include the following: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.★ (A2, M3) b. Focus on linear, quadratic, and exponential functions. (A1, M2)
F.IF.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.★ b. Focus on linear, quadratic, and exponential functions. (A1, M2)

Analyze functions using different representations.

F.IF.7	Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. ★ a. Graph linear functions and indicate intercepts. (A1, M1) b. Graph quadratic functions and indicate intercepts, maxima, and minima. (A1, M2) e. Graph simple exponential functions, indicating intercepts and end behavior. (A1, M1)
F.IF.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. (A2, M3) i. Focus on completing the square to quadratic functions with the leading coefficient of 1. (A1) b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of changeG in functions such as y = (1.02)t , and y = (0.97)t and classify them as representing exponential growth or decay. (A2, M3) i. Focus on exponential functions evaluated at integer inputs. (A1, M2)
F.IF.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. (A2, M3) b. Focus on linear, quadratic, and exponential functions. (A1, M2)

Building Functions

Build a function that models a relationship between two quantities.

F.BF.1	Write a function that describes a relationship between two quantities. ★ a. Determine an explicit expression, a recursive process, or steps for calculation from context. i. Focus on linear and exponential functions. (A1, M1) ii. Focus on situations that exhibit quadratic or exponential relationships. (A1, M2)
F.BF.2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. ★

Build new functions from existing functions.

F.BF.3	Identify the effect on the graph of replacing f(x) by f(x) + k, kf(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. (A2, M3) a. Focus on transformations of graphs of quadratic functions, except for f(kx); (A1, M2)
F.BF.4	Find inverse functions. a. Informally determine the input of a function when the output is known. (A1, M1)

Linear, Quadratic, and Exponential Models

Construct and compare linear, quadratic, and exponential models, and solve problems.

F.LE.1	Distinguish between situations that can be modeled with linear functions and with exponential functions. ★ a. Show that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another
F.LE.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). ★
F.LE.3	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically. ★(A1, M2)

Interpret expressions for functions in terms of the situation they model.

F.LE.5	Interpret the parameters in a linear or exponential function in terms of a context. ★

Interpreting Categorical and Quantitative Data

Summarize, represent, and interpret data on a single count or measurement variable.

S.ID.1	Represent data with plots on the real number line (dot plots, histograms, and box plots) in the context of real-world applications using the GAISE model. ★
S.ID.2	In the context of real-world applications by using the GAISE model, use statistics appropriate to the shape of the data distribution to compare center (median and mean) and spread (mean absolute deviationG, interquartile range, and standard deviation) of two or more different data sets. ★
S.ID.3	In the context of real-world applications by using the GAISE model, interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). ★
S.ID.5	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. ★
S.ID.6	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. ★ c. Fit a linear function for a scatterplot that suggests a linear association. (A1, M1)

Interpret expressions for functions in terms of the situation they model.

S.ID.7	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. ★
S.ID.6	Compute (using technology) and interpret the correlation coefficient of a linear fit. ★

Algebra 1 Scope and Sequence

	Number & Quantities	Seeing Structure in Expressions	Arithmetic with Polynomials and Rational Expressions	Creating Equations	Reasoning with Equations & Inequalities	Interpreting Functions	Building Functions	Linear, Quadratic & Exponential Models	Interpreting Categorical & Quantitative Data
SEMESTER 1									
Unit 1 • Representing Relationships Mathematically	N.Q.1 N.Q.2 N.Q.3	A.SSE.1a		A.CED.1a A.CED.4a	A.REI.1 A.REI.3			F.LE.1a	
Unit 2 • Understanding Functions	N.Q.1 N.Q.2 N.Q.3					8.F.1 F.IF.1 8.F.2 F.IF.2 8.F.5 F.IF.3 F.IF.5b F.IF.9b		F.LE.1a	
Unit 3 • Graphing Linear Functions & Inequalities	N.Q.1 N.Q.2 N.Q.3			A.CED.3a A.CED.4a	A.REI.12	8.F.3 8.F.4 8.F.5	F.BF.4a F.BF.5b F.BF.6	F.LE.1a,b F.LE.5b F.LE.7a F.LE.9	8.SP.3 S.ID.7
Unit 4 • Writing Linear Functions & Inequalities	N.Q.1 N.Q.2 N.Q.3			A.CED.3			F.BF.2	F.LE.2	8.SP.1 8.SP.2 8.SP.3 8.SP.4 S.ID.6c S.ID.7 S.ID.8
Unit 5 • Systems of Equations & Inequalities	N.Q.1 N.Q.2 N.Q.3			A.CED.2a A.CED.3 8.EE.7a 8.EE.8a,b,c	A.REI.5 A.REI.6a A.REI.11 A.REI.12				
SEMESTER 2									
Unit 6 • Polynomial Expressions & Equations	N.Q.1 N.Q.2 N.Q.3	A.SSE.1b A.SSE.2 A.SSE.3a	A.APR.1a A.APR.3	A.CED.1b					
Unit 7 • Graphing Quadratic Functions	N.Q.1 N.Q.2 N.Q.3	A.SSE.3a		A.CED.2b		F.IF.4b F.IF.7b F.IF.8a	F.BF.3a		
Unit 8 • Solving Quadratic Equations	8.NS.2 N.Q.1 N.Q.2 N.Q.3	A.SSE.3a,b		A.CED.3 8.EE.2	A.REI.4a,b A.REI.7 A.REI.11	F.IF.8a			
Unit 9 • Exponential Functions	N.Q.1 N.Q.2 N.Q.3	A.SSE.1b A.SSE.3c		A.CED.2a 8.EE.2		F.IF.7e F.IF.8b	F.BF.1a F.BF.2	F.LE.1a,b,c F.LE.2 F.LE.3 F.LE.5	
Unit 10 • Statistical Models	N.Q.1 N.Q.2 N.Q.3								8.SP.4 S.ID.1 S.ID.2 S.ID.3 S.ID.5

Geometry Standards
(Courses: Geometry, Honors Geometry, Mathematical Modeling & Reasoning, Algebra 3, PreCalculus)

Congruence

Experiment with transformations in the plane.

G.CO.1	Know precise definitions of ray, angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and arc length.
G.CO.2	Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not, e.g., translation versus horizontal stretch.
G.CO.3	Identify the symmetries of a figure, which are the rotations and reflections that carry it onto itself. a. Identify figures that have line symmetry; draw and use lines of symmetry to analyze properties of shapes. b. Identify figures that have rotational symmetry; determine the angle of rotation, and use rotational symmetry to analyze properties of shapes.
G.CO.4	Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
G.CO.5	Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using items such as graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Understand congruence in terms of rigid motions.

G.CO.6	Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
G.CO.7	Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
G.CO.8	Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

Prove geometric theorems both formally and informally using a variety of methods.

G.CO.9	Prove and apply theorems about lines and angles. Theorems include but are not restricted to the following: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.
G.CO.10	Prove and apply theorems about triangles. Theorems include but are not restricted to the following: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

G.CO.11	Prove and apply theorems about parallelograms. Theorems include but are not restricted to the following: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

Make geometric constructions

G.CO.12	Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.
G.CO.13	Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Classify and analyze geometric figures.

G.CO.14	Classify two-dimensional figures in a hierarchy based on properties

Similarity, Right Triangles, and Trigonometry

Understand similarity in terms of similarity transformations.

G.SRT.1	Verify experimentally the properties of dilations given by a center and a scale factor: a. A dilation takes a line not passing through the center of the dilation to a parallel line and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
G.SRT.2	Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
G.SRT.3	Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Prove and apply theorems both formally and informally involving similarity using a variety of methods.

G.SRT.4	Prove and apply theorems about triangles. Theorems include but are not restricted to the following: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.
G.SRT.5	Use congruence and similarity criteria for triangles to solve problems and to justify relationships in geometric figures that can be decomposed into triangles.

Define trigonometric ratios, and solve problems involving right triangles.

G.SRT.6	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

G.SRT.7	Explain and use the relationship between the sine and cosine of complementary angles.
G.SRT.8	Solve problems involving right triangles. ★ a. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems if one of the two acute angles and a side length is given. (G, M2)

Circles

Understand and apply theorems about circles.

G.C.1	Prove that all circles are similar using transformational arguments.
G.C.2	Identify and describe relationships among angles, radii, chords, tangents, and arcs and use them to solve problems. Include the relationship between central, inscribed, and circumscribed angles and their intercepted arcs; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.
G.C.3	Construct the inscribed and circumscribed circles of a triangle; prove and apply the property that opposite angles are supplementary for a quadrilateral inscribed in a circle.
(+) G.C.4	Construct a tangent line from a point outside a given circle to the circle.

Find arc lengths and areas of sectors of circles.

G.C.5	Find arc lengths and areas of sectors of circles. a. Apply similarity to relate the length of an arc intercepted by a central angle to the radius. Use the relationship to solve problems. b. Derive the formula for the area of a sector, and use it to solve problems.

Expressing Geometric Properties with Equations

Translate between the geometric description and the equation for a conic section.

G.GPE.1	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

Use coordinates to prove simple geometric theorems algebraically and to verify specific geometric statements.

G.GPE.4	Use coordinates to prove simple geometric theorems algebraically and to verify geometric relationships algebraically, including properties of special triangles, quadrilaterals, and circles. For example, determine if a figure defined by four given points in the coordinate plane is a rectangle; determine if a specific point lies on a given circle. (G, M2)
G.GPE.5	Justify the slope criteria for parallel and perpendicular lines, and use them to solve geometric problems, e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point.

G.GPE.6	Justify the slope criteria for parallel and perpendicular lines, and use them to solve geometric problems, e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point.
G.GPE.7	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula. ★

Geometric Measurement and Dimension

Explain volume formulas, and use them to solve problems.

G.GMD.1	Give an informal argument for the formulas for the circumference of a circle, area of a circle, and volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.
G.GMD.3	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. ★

Visualize relationships between two-dimensional and three-dimensional objects.

G.GMD.4	Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Understand the relationships between lengths, areas, and volumes.

G.GMD.5	Understand how and when changes to the measures of a figure (lengths or angles) result in similar and non-similar figures.
G.GMD.6	When figures are similar, understand and apply the fact that when a figure is scaled by a factor of k, the effect on lengths, areas, and volumes is that they are multiplied by k, k^2 , and k^3 , respectively.

Modeling with Geometry

Apply geometric concepts in modeling situations.

G.MG.1	Use geometric shapes, their measures, and their properties to describe objects, e.g., modeling a tree trunk or a human torso as a cylinder. ★
G.MG.2	Apply concepts of density based on area and volume in modeling situations, e.g., persons per square mile, BTUs per cubic foot. ★
G.MG.3	Apply geometric methods to solve design problems, e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios. ★

Conditional Probability and Rules of Probability

Understand independence and conditional probability, and use them to interpret data.

S.CP.1	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”). ★

S.CP.2	Understand that two events A and B are independent if and only if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent. ★
S.CP.3	Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B. ★
S.CP.4	Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results. ★
S.CP.5	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer. ★

Use the rules of probability to compute probabilities of compound events in a uniform probability model.

S.CP.6	Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model. ★
S.CP.7	Apply the Addition Rule, P(A or B) = P(A) + P(B) - P(A and B), and interpret the answer in terms of the model. ★
(+) S.CP.8	Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)•P(B|A) = P(B)•P(A|B), and interpret the answer in terms of the model.★ (G, M2)
(+) S.CP.9	Use permutations and combinations to compute probabilities of compound events and solve problems. ★ (G, M2)

Geometry & Honors Geometry* Scope and Sequence

(* Denotes additional standards for Honors Geometry)

	Congruence	Similarity, Right Triangles, & Trigonometry	Circles	Expressing Geometric Properties with Equations	Geometric Measurements & Dimension	Model with Geometry	Conditional Probability
SEMESTER 1							
Unit 1 • Essentials of Geometry	G.CO.1 G.CO.12			G.GPE.4 G.GPE.6 G.GPE.7			
Unit 2 • Reasoning & Proof	G.CO.9						
Unit 3 • Parallel & Perpendicular Lines	G.CO.1 G.CO.9 G.CO.12			G.GPE.4 G.GPE.5			
Unit 4 • Rigid Transformations & Congruence	G.CO.2 G.CO.3 G.CO.4 G.CO.5 G.CO.6 G.CO.7 G.CO.8 G.CO.9 G.CO.12	G.SRT.5		G.GPE.4 G.GPE.5			
Unit 5 • Triangles & Their Properties	G.CO.10 G.CO.12	G.SRT.4 G.SRT.5	G.C.3				
SEMESTER 2							
Unit 6 • Dilations & Similarity	G.CO.2	G.SRT.1 G.SRT.2 G.SRT.3 G.SRT.4 G.SRT.5			G.GMD.5 G.GMD.6		
Unit 7 • Right Triangle Trigonometry	G.SRT.4 G.SRT.5 G.SRT.6 G.SRT.7 G.SRT.8					G.MG.1	
Unit 8 • Circles	G.CO.1 G.CO.13	G.SRT.5	G.C.1 G.C.2 G.C.3 G.C.4 (+) G.C.5	G.GPE.1 G.GPE.4	G.GMD.1	G.MG.1	
Unit 9 • Conditional Probability							S.CP.1 S.CP.2 S.CP.3 S.CP.4 S.CP.5 S.CP.6 S.CP.7 S.CP.8 (+) S.CP.9 (+)
Unit 10 • Area and Perimeter of Polygons	G.CO.3 G.CO.11 G.CO.14		G.SRT.5	G.GPE.4 G.GPE.5 G.GPE.7	G.GMD.5 G.GMD.6	G.MG.1 G.MG.3	
Unit 11 • Extending Surface Area & Volume					G.GMD.1 G.GMD.3 G.GMD.4 G.GMD.6	G.MG.1 G.MG.2 G.MG.3	

Algebra 2 Standards
(Courses: Algebra 2, Honors Algebra 2, Mathematical Modeling and Reasoning, Data Science Foundations, Algebra 3, Precalculus)

The Real Number System

Extend the properties of exponents to rational exponents.

N.RN.1	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define 51/3 to be the cube root of 5 because we want (51/3)3 = 5(1/3)3 to hold, so (51/3)3 must equal 5.
N.RN.2	Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Use properties of rational and irrational numbers.

N.RN.3	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

The Complex Number System

Perform arithmetic operations with complex numbers.

N.CN.1	Know there is a complex number i such that iˆ2 = -1, and every complex number has the form a + bi with a and b real.
N.CN.2	Use the relation iˆ2= -1 and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
N.CN.3	Find the conjugate of a complex number; use conjugates to find magnitudes and quotients of complex numbers.

Use complex numbers in polynomial identities and equations

N.CN.7	Solve quadratic equations with real coefficients that have complex solutions.
(+) N.CN.8	Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$.
(+) N.CN.9	Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Seeing Structure in Expressions

Interpret the structure of expressions

A.SSE.1	Interpret expressions that represent a quantity in terms of its context. ★ a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity.

A.SSE.2	Use the structure of an expression to identify ways to rewrite it. For example, to factor $3x(x - 5) + 2(x - 5)$, students should recognize that the "x - 5" is common to both expressions being added, so it simplifies to $(3x + 2)(x - 5)$; or see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

Write expressions in equivalent forms to solve problems.

A.SSE.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. ★ c. Use the properties of exponents to transform expressions for exponential functions. For example, 8^t can be written as 2^{3t}
(+) A.SSE.4	Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments. ★

Arithmetic with Polynomials and Rational Expressions

Perform arithmetic operations on polynomials.

A.APR.1	Understand that polynomials form a system analogous to the integers, namely, that they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. b. Extend to polynomial expressions beyond those expressions that simplify to forms that are linear or quadratic. (A2, M3)

Understand the relationship between zeros and factors of polynomials.

A.APR.2	Understand and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x - a is p(a). In particular, p(a) = 0 if and only if (x – a) is a factor of p(x).
A.APR.3	Identify zeros of polynomials, when factoring is reasonable, and use the zeros to construct a rough graph of the function defined by the polynomial.

Use polynomial identities to solve problems.

(+) A.APR.5	Know and apply the Binomial Theorem for the expansion of (x + y)n in powers of x and y for a positive integer n, where x and y are any numbers. For example, by using coefficients determined by Pascal's Triangle. The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.

Rewrite rational expressions.

A.APR.6	Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.
(+) A.APR.7	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Creating Equations

Create equations that describe numbers or relationships.

A.CED.1	Create equations and inequalities in one variable and use them to solve problems. Include equations and inequalities arising from linear, quadratic, simple rational, and exponential functions. ★ c. Extend to include more complicated function situations with the option to solve with technology. (A2, M3)
A.CED.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. ★ c. Extend to include more complicated function situations with the option to graph with technology. (A2, M3)
A.CED.3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods. ★ (A1, M1) a. While functions will often be linear, exponential, or quadratic, the types of problems should draw from more complicated situations. (A2, M3)
A.CED.4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. ★ d. While functions will often be linear, exponential, or quadratic, the types of problems should draw from more complicated situations. (A2, M3)

Reasoning with Equations and Inequalities

Understand solving equations as a process of reasoning and explain the reasoning.

A.REI.2	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Solve systems of equations.

A.REI.6	Solve systems of linear equations algebraically and graphically. b. Extend to include solving systems of linear equations in three variables, but only algebraically. (A2, M3)

Represent and solve equations and inequalities graphically.

A.REI.11	Explain why the x-coordinates of the points where the graphs of the equation $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, making tables of values, or finding successive approximations.

Interpreting Functions

Interpret functions that arise in applications in terms of the context.

F.IF.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include the following: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. ★ (A2, M3)

F.IF.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. ★ c. Emphasize the selection of a type of function for a model based on behavior of data and context. (A2, M3)
F.IF.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★ (A2, M3)

Analyze functions using different representations.

F.IF.7	Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. c. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. (A2, M3) d. Graph polynomial functions, identifying zeros, when factoring is reasonable, and indicating end behavior. (A2, M3) f. Graph exponential functions, indicating intercepts and end behavior, and trigonometric functions, showing period, midlineG, and amplitude. (A2, M3) (+) g. Graph rational functions, identifying zeros and asymptotes when factoring is reasonable, and indicating end behavior. (A2, M3) (+) h. Graph logarithmic functions, indicating intercepts and end behavior. (A2, M3)
F.IF.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. (A2, M3) b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as y = (1.02)t , and y = (0.97)t and classify them as representing exponential growth or decay. (A2, M3)
F.IF.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. (A2, M3) b. Focus on linear, quadratic, and exponential functions. (A1, M2)

Building Functions

Build new functions from existing functions.

F.BF.1	Compose functions. ★ b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. (A2, M3) (+) c. Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.

F.BF.3	Identify the effect on the graph of replacing f(x) by f(x) + k, kf(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. (A2, M3)
F.BF.4	Find inverse functions. (+) b. Read values of an inverse function from a graph or a table, given that the function has an inverse. (A2, M3) (+) c. Verify by composition that one function is the inverse of another. (A2, M3) (+) d. Find the inverse of a function algebraically, given that the function has an inverse. (A2, M3)

Linear, Quadratic, and Exponential Models

Construct and compare linear, quadratic, and exponential models, and solve problems.

F.LE.4	For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. ★

Geometry

Define trigonometry ratios, and solve problems involving right triangles.

G.SRT.8 (+)	Solve problems involving right triangles. ★ (+) b. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★ (A2, M3)

Apply trigonometry to general triangles.

G.SRT.9 (+)	Derive the formula *A=1/2 absin(C)* for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
G.SRT.10 (+)	Explain proofs of the Laws of Sines and Cosines and use the Laws to solve problems.
G.SRT.11 (+)	Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles, e.g., surveying problems, resultant forces.

Statistics and Probability

Summarize, represent, and interpret data on two categorical and quantitative variables.

S.ID.6	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.★ a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions, or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. (A2, M3) b. Informally assess the fit of a function by discussing residuals. (A2, M3)

Algebra 2 & Honors Algebra 2 Scope and Sequence

(* Denotes additional standards for Honors Algebra 2)

	The Real Number System	The Complex Number System	Seeing Structure in Expressions	Creating Equations	Reasoning with Equations & Inequalities	Arithmetic with Polynomials & Rational Expressions	Interpreting Functions	Building Functions	Linear, Quadratic & Exponential Models	Similarity, Right Triangles, & Trigonometry	Interpreting Categorical & Quantitative Data
SEMESTER 1											
Unit P Algebra Review				A.CED.1a A.CED.2a A.CED.4	A.REI.3, A.REI.10		F.IF.6,10		F.LE.2		S.ID.6a,b,c S.ID.7 S.ID.8
Unit 1 • Foundations of Functions					A.REI.3		F.IF.1 F.IF.2 F.IF.5 F.IF.7c	F.BF.3			
Unit 2 • Quadratic Functions	N.RN.2 N.RN.3	N.CN.1 N.CN.2 N.CN.3 N.CN.7	A.SSE 1a,b A.SSE.2 A.SSE.3a,b	A.CED.1b	A.REI.4a,b		F.IF.4 F.IF.7a F.IF.8a F.IF.9	F.BF.3			S.ID.6a
Unit 3 • Systems of Linear & NonLinear				A.CED.2c A.CED.3c	A.REI.6 A.REI.7 A.REI.11 A.REI.7						
SEMESTER 2											
Unit 4 • Polynomial Functions		N.CN.3 N.CN.7 N.CN.8(+) N.CN.9(+)	A.SSE 1a,b A.SSE.2		A.REI.4b	A.APR.1b A.APR.2 A.APR.3 A.APR.5(+) A.APR.6	F.IF.2 F.IF.4 F.IF.7d F.IF.8a	F.BF.3			
Unit 5 • Radical Functions	N.RN.1 N.RN.2		A.SSE.2		A.REI.2		F.IF.4 F.IF.5 F.IF.7c	F.BF.1b,c F.BF.3 F.BF.4b-d			
Unit 6 • Exponential & Logarithmic Functions			A.SSE.2 A.SSE.3c A.SSE.4(+)	A.CED.1c A.CED.2c			F.IF.4 F.IF.5c F.IF.7f,h F.IF.8b	F.BF.3 F.BF.5(+)	F.LE.2 F.LE.4		S.ID.6a,b
Unit 7 • Rational Functions			A.SSE.2	A.CED.1c A.CED.2c	A.REI.2	A.APR.6 A.APR.7(+)	F.IF.4 F.IF.5c F.IF.7g(+)	F.BF.3			
Unit 9 • Exponential Functions										G.SRT.6 G.SRT.8 G.SRT.9 G.SRT.10 G.SRT.11	

Statistics & Probability Standards
(Courses: Geometry, Mathematical Modeling & Reasoning, Data Science Foundations, Statistics, AP Statistics)

Interpret Categorical and Quantitative Data

Summarize, represent, and interpret data on a single count or measurement variable.

S.ID.1	Represent data with plots on the real number line (dot plots, histograms, and box plots) in the context of real world applications using the GAISE model. ★
S.ID.2	In the context of real-world applications by using the GAISE model, use statistics appropriate to the shape of the data distribution to compare center (median and mean) and spread (mean absolute deviationG, interquartile rangeG, and standard deviation) of two or more different data sets. ★
S.ID.3	In the context of real-world applications by using the GAISE model, interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). ★
S.ID.4	Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. ★

Interpret Linear Models

S.ID.7	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. ★
S.ID.8	Compute (using technology) and interpret the correlation coefficient of a linear fit. ★
S.ID.9 (+)	Distinguish between correlation and causation. ★

Recognize and interpret the normal distribution model.

S.ID.10	Understand discrete and continuous distributions and distinguish between them. Compare to uniform continuous, uniform discrete, binomial, and normal distributions. ★ a. Calculate probabilities for various distributions. b. Use one kind of distribution to approximate the other.
S.ID.11	Visually compare a data distribution to the standard normal distribution. Recognize that normal distributions can be used to represent some population distributions. Understand that a normal distribution is determined by its mean and standard deviation. ★
S.ID.12	Calculate and use probability from a normal distribution. ★ a. Determine proportions and percentiles from a normal distribution. Understand that the 50th percentile is a measure-of-center, the median. b. Compare measures of relative position in data sets: z-scores and percentiles.
S.ID.13	Use the standard normal distribution to approximate binomial distributions. ★

Interpret Categorical and Qualitative Data

Understand and evaluate random processes underlying statistical experiments

S.IC.1	Understand statistics as a process for making inferences about population parameters based on a random sample from that population. ★
S.IC.2	Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model? ★

Make inferences and justify conclusions from sample surveys, experiments, and observational studies

S.IC.3	Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. ★
S.IC.4	Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. ★
S.IC.5	Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between sample statistics are statistically significant.
S.IC.6	Evaluate reports based on data. ★

Conditional Probability and Rules of Probability

Understand independence and conditional probability, and use them to interpret data.

S.CP.1	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). ★
S.CP.2	Understand that two events A and B are independent if and only if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent. ★
S.CP.3	Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B. ★
S.CP.4	Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.★
S.CP.5	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer. ★

Use the rules of probability to compute probabilities of compound events in a uniform probability model.

S.CP.6	Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model. ★
S.CP.7	Apply the Addition Rule, P(A or B) = P(A) + P(B) - P(A and B), and interpret the answer in terms of the model. ★
(+) S.CP.8	Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)•P(B|A) = P(B)•P(A|B), and interpret the answer in terms of the model. ★ (G, M2)
(+) S.CP.9	Use permutations and combinations to compute probabilities of compound events and solve problems. ★ (G, M2)

Use the rules of probability to compute probabilities of compound events in a uniform probability model.

(+)S.CP.9	Use permutations and combinations to compute probabilities of compound events and solve problems. ★

Using Probability To Make Decisions

Calculate expected values, and use them to solve problems.

S.MD.1	Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distributionG using the same graphical displays as for data distributions. ★
(+) S.MD.2	Calculate the expected valueG of a random variable; interpret it as the mean of the probability distribution. ★
(+) S.MD.3	Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes. ★
S.MD.4	Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households? ★

Use probability to evaluate outcomes of decisions.

(+) S.MD.5	Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. ★ a. Find the expected payoff for a game of chance. For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant. b. Evaluate and compare strategies on the basis of expected values. For example, compare a high-deductible versus a low deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.
(+) S.MD.6	Use probabilities to make fair decisions, e.g., drawing by lots, using a random number generator. ★
(+) S.MD.7	Analyze decisions and strategies using probability concepts, e.g., product testing, medical testing, pulling a hockey goalie at the end of a game. ★

Statistics Scope & Sequence

	Interpreting Categorical & Quantitative Data	Making Inferences & Justifying Conclusions	Conditional Probability	Using Probability to Make Decisions
SEMESTER 1				
Unit 1 • Interpreting Univariate Data	S.ID.1 S.ID.2 S.ID.3 S.ID.4			
Unit 2 • Interpret Bivariate Data	S.ID.5 S.ID.6a,b,c S.ID.7 S.ID.8 S.ID.9 (+)			
Unit 3 • Probability Distributions			S.CP.1 S.CP.3 S.CP.4 S.CP.5 S.CP.6 S.CP.7 S.CP.8 (+) S.CP.9 (+)	S.MD.1 S.MD.2 S.MD.3(+) S.MD.4(+) S.MD.5a,b (+) S.MD.6(+) S.MD.7(+)
SEMESTER 2				
Unit 4 • Collecting Data, Making Inferences, and Justifying Conclusions from Data		S.IC.1 S.IC.2 S.IC.3		
Unit 5 • Significance Testing		S.IC.5 S.IC.6		
Unit 8 • Project-Based Learning	All of the standards above are used in this final culminating unit.			

Mathematical Modeling & Reasoning Scope & Sequence

	Number & Quantity	Model with Geometry/ Geometric Measurements & Dimension	Congruence/ Circles	Seeing Structure in Expressions	Creating Equations	Reasoning with Equations & Inequalities	Arithmetic with Polynomials & Rational Expressions	Interpreting Functions	Building Functions	Linear, Quadratic & Exponential Models/Trigonometric Functions	Similarity, Right Triangles, and Trigonometry	Interpreting Categorical & Quantitative Data
SEMESTER 1												
Theme #1 • Number & Quantity	N.Q.3 N.Q.1 N.Q.2	G.MG.3			A.CED.1a				F.BF.1			S.ID.9
Theme #2 • Functions - Part 1				A.SSE.1 A.SSE.3	A.CED.2a A.CED.3 A.CED.2b A.CED.2c	A.REI.6 A.REI.10	A.APR.3	F.IF.4 F.IF.6	F.BF.1a	F.LE.1 F.LE.3 F.LE.5		S.ID.6 S.ID.8 S.ID.9
Theme #3 • Functions - Part 2	N.Q.1 N.Q.2 N.Q.3			A.SSE.1 A.SSE.4	A.CED.1 A.CED.2c A.CED.3 A.CEd.4	A.REI.1 A.REI.4b A.REI.10 A.REI.12		F.IF.1 F.IF.3 F.IF.4 F.IF.5b,c F.IF.7 F.IF.8 F.IF.9	F.BF.1 F.BF.2	F.LE.1 F.LE.2 F.LE.3 F.LE.5		S.ID.6
Theme #4 • Geometry	G.MG.3 G.GMD.1	G.CO.2 G.CO.3 G.CO.5 G.CO.12 G.C.5						F.IF.7	F.BF.1	F.TF.2 F.TF.5	G.SRT.4 G.SRT.5	

SEMESTER 2

	Creating Equations	Using Probability to Make Decisions	Conditional Probability	Interpreting Categorical & Quantitative Data	Making Inferences & Justifying Conclusions
Theme #5 • Statistics				S.ID.1 S.ID.2 S.ID.3 S.ID.4 S.ID.6 S.ID.7 S.ID.8 S.ID.9	S.IC.1 S.IC.3 S.IC.4 S.IC.5 S.IC.6
Theme #6 • Probability		S.MD.1 S.MD.3 S.MD.4 S.MD.5 S.MD.7	S.CP.1 S.CP.3 S.CP.4 S.CP.5 S.CP.6		S.IC.2 S.IC.3
Theme #3 • Applications of Number & Quantity, & Statistics	A.CED.1c A.CED.2c A.CED.3c A.CED.4d				S.IC.1 S.IC.6

Data Science Foundations Scope & Sequence

	Number & Quantity/Vector & Matrix Quantities	Creating & Reasoning with Equations	Interpreting Functions	Building Functions	Linear, Quadratic & Exponential Models	Interpreting Categorical & Quantitative Data	Making Inferences & Justifying Conclusions	Conditional Probability/Using Probability to Make Decisions
SEMESTER 1								
Unit #1 • Interpreting Data	N.Q.1 N.Q.2 N.Q.3					S.ID.1 S.ID.2 S.ID.3 S.ID.5 S.ID.6	S.IC.6	
Unit #2 • Making Inferences & Justifying Conclusions	N.Q.1 N.Q.2 N.Q.3					S.ID.2 S.ID.3 S.ID.4	S.IC.2 S.IC.6	S.CP.2 S.CP.9 (+)
SEMESTER 1								
Unit #3 • Probability	N.Q.1 N.Q.2 N.Q.3						S.IC.1 S.IC.3 S.IC.6	S.CP.4 S.CP.5 S.MD.7
Unit #4 • Algebra in RStudio	N.Q.1 N.Q.2 N.Q.3 N.VM.6(+)	A.CED.1 A.CED.2 A.CED.3 A.CED.4	F.IF.1 F.IF.2 F.IF.4 F.IF.5c F.IF.6	F.BF.1a,b,c	F.LE.1 F.LE.2 F.LE.5	S.ID.6a,b,c S.ID.7 S.ID.8 S.ID.9 (+)	S.IC.2 S.IC.6	

Fourth Course Standards
(Courses: Mathematical Modeling & Reasoning,
Data Science Foundations, Algebra 3, PreCalculus, Statistics)

Vector and Matrix Quantities

Represent and model with vector quantities.

(+)N.VM.1	Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes, e.g., v, $\lvert v \rvert$, $\lVert v \rVert$, $\vec{v}$.
(+)N.VM.2	Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
(+)N.VM.3	Solve problems involving velocity and other quantities that can be represented by vectors.

Perform operations on vectors.

(+)N.VM.4	Add and subtract vectors. a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. c. Understand vector subtraction $v - w$ as $v + (-w)$, where $-w$ is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise
(+)N.VM.5	Multiply a vector by a scalar. a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.

Perform operations on matrices, and use matrices in applications.

(+)N.VM.6	Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
(+)N.VM.7	Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
(+)N.VM.8	Add, subtract, and multiply matrices of appropriate dimensions.
(+)N.VM.9	Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties
(+)N.VM.10	Understand that the zero and identity matrices play a role in matrix addition and multiplication analogous to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
(+)N.VM.11	Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
(+)N.VM.12	Work with 2 x 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

Arithmetic with Polynomials and Rational Expressions

Use polynomial identities to solve problems

A.APR.4	Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $(x^2 + y^2)2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.
(+)A.APR.5	Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers. For example by using coefficients determined by Pascal's Triangle. The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.

Rewrite rational expressions.

A.APR.6	Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.
(+)A.APR.7	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Reasoning with Equations and Inequalities

Solve systems of equations.

(+)A.REI.8	Represent a system of linear equations as a single matrix equation in a vector variable
(+)A.REI.9	Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 x 3 or greater).

Building Functions

Build new functions from existing functions.

F.BF.4	Find inverse functions. (+) b. Read values of an inverse function from a graph or a table, given that the function has an inverse. (A2, M3) (+) c. Verify by composition that one function is the inverse of another. (A2, M3) (+) d. Find the inverse of a function algebraically, given that the function has an inverse. (A2, M3) (+) e. Produce an invertible function from a non-invertible function by restricting the domain.
(+)F.BF.5	Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Linear, Quadratic, and Exponential Models

Interpret expressions for functions in terms of the situation they model.

F.LE.5	Interpret the parameters in a linear or exponential function in terms of a context. ★

Trigonometric Functions

Extend the domain of trigonometric functions using the unit circle.

F.TF.1	Understand the radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
F.TF.2	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
(+)F.TF.3	Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$, and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.
(+)F.TF.4	Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

F.TF.5	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. ★
(+)F.TF.6	Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
(+)F.TF.7	Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. ★

Prove and apply trigonometric identities

F.TF.8	Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$, and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.
(+)F.TF.9	Prove the addition and subtraction formulas for sine, cosine, and tangent, and use them to solve problems.

Circles

Find arc lengths and areas of sectors of circles

G.C.5	Find arc lengths and areas of sectors of circles. a. Apply similarity to relate the length of an arc intercepted by a central angle to the radius. Use the relationship to solve problems. b. Derive the formula for the area of a sector, and use it to solve problems.

G.C.6	Derive formulas that relate degrees and radians, and convert between the two. (A2, M3)

Expressing Geometric Properties with Equations

Translate between the geometric description and the equation for a conic section.

G.GPE.1	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation
(+)G.GPE.2	Derive the equation of a parabola given a focus and directrix.
(+)G.GPE.3	Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

Algebra 3 Scope & Sequence

	Vector & Matrix Quantities	The Complex Number System	Seeing Structure in Expressions	Creating Equations	Reasoning with Equations & Inequalities	Arithmetic with Polynomials & Rational Expressions	Interpreting & Building Functions	Linear, Quadratic & Exponential Models	Circle/ Similarity, Right Triangles, and Trigonometry	Trigonometric Functions
SEMESTER 1										
Unit 1 • Matrices & Systems	(+)N.VM.6 (+)N.VM.7 (+)N.VM.8 (+)N.VM.9 (+)N.VM.10			A.CED.2 A.CED.3a	A.REI.5 A.REI.6 A.REI.7 A.REI.8(+) A.REI.9(+)					
Unit 2 • Foundations of Functions							F.BF.3		G.C.6 G.SRT.6	F.TF.1 F.TF.2 F.TF.3(+) F.TF.4(+) F.TF.5 F.TF.7(+) F.TF.8(+)
Unit 3 • Applications with Trigonometry									G.SRT.8a,b G.SRT.9 G.SRT.10 G.SRT.11	
Unit 4 • Solving Equations & Inequalities		N.CN.1 N.CN.2 N.CN.3 N.CN.7 N.CN.8 N.CN.9	A.SSE.2 A.SSE.3a	A.CED.1 A.CED.3 A.CED.4	A.REI.2 A.REI.4 A.REI.11		F.IF.4			
SEMESTER 2										
Unit 5 • Functions & Their Graphs			A.SSE.3a	A.CED.2a			F.IF.1 F.IF.2 F.IF.4 F.IF.5 F.IF.6 F.IF.7a,c F.IF.9c F.BF.3 F.BF.4a,b,c,d	F.LE.2		
Unit 6 • Polynomial Functions		N.CN.7 N.CN.8 N.CN.9		A.CED.2b,c		A.APR.2 A.APR.3 A.APR.6	F.IF.4 F.IF.5 F.IF.7b,d F.IF.8 F.IF.9 F.BF.3			
Unit 7 • Rational Functions			A.SSE.2		A.REI.2	A.APR.6 A.APR.7(+)	F.IF.4 F.IF.5 F.IF.7g			
Unit 8 • Exponential & Logarithmic Functions				A.CED.1 A.CED.2c			F.IF.4 F.IF.5 F.IF.7f,h F.IF.8 F.BF.3 F.BF.5(+)	F.LE.2 F.LE.4		

PreCalculus Scope & Sequence

	The Complex Number System	Seeing Structure in Expressions	Reasoning with Equations & Inequalities	Arithmetic with Polynomials & Rational Expressions	Interpreting & Building Functions	Linear, Quadratic & Exponential Models	Creating Equations	Conditional Probability	Interpreting Categorical & Quantitative Data
SEMESTER 1									
Unit 1 • Parent Functions			A.REI.11		F.IF.1 F.IF.,2 F.IF.,4 F.IF.5 F.IF.7c F.BF.1b,c, F.BF.3, F.BF.4a,b,c				S.ID.6a
Unit 2 • Polynomial & Rational Functions	N.CN.3(+) N.CN.7 N.CN.8(+) N.CN.9(+) G.SRT.8a	A.SSE.1 A.SSE.3a,b	A.REI.2 A.REI.11	A.APR.2 A.APR.3 A.APR.6	F.IF.4 F.IF.6 F.IF.7a,b,c,d,g(+) F.BF.1a		A.CED.1c		S.ID.6 S.ID.7 S.ID.8 S.ID.9
Unit 3 • Exponential & Logarithmic Functions		A.SSE.4(+)			F.IF.4b F.IF.5 F.IF.7f, h F.IF.8b F.BF.5	F.LE.2 F.LE.4			
Unit 9 • Discrete Math		A.SSE.4(+)		A.APR.5	A.F.IF.3 F.BF.2			S.CP..9(+)	

SEMESTER 2	The Complex Number System	Vector & Matrix Quantities	Reasoning with Equations & Inequalities	Interpreting & Building Functions	Circle/Similarity, Right Triangles, and Trigonometry	Linear, Quadratic & Exponential Models	Creating Equations	Trigonometric Functions	Expressing Geometric Properties with Equations
Unit 4 • Trigonometric Functions				F.IF.7f	G.C.5b G.C.6 G.SRT.6 G.SRT.8a,b(+)			F.TF.1 F.TF.2 (+) F.TF.3 (+) F.TF.4) F.TF.5 (+) F.TF.6 (+) F.TF.7	
Unit 5 • Analytical Trigonometry					G.SRT,9 G.SRT.10 G.SRT.11(+)			F.TF.7, 8, 9(+)	
Unit 6 • Applications of Trigonometry	N.CN.4(+) N.CN.5(+) N.CN.6	(+)N.VM.1 (+)N.VM.2 (+)N.VM.3 (+)N.VM.4a,b,c (+)N.VM.5a,b		F.IF.4 F.IF.5 F.IF.7f,h F.IF.8 F.BF.3 F.BF.5		F.LE.2 F.LE.4			
Unit 7 • Systems & Matrices		(+) N.VM.6 (+)N.VM.7 (+)N.VM.8 (+)N.VM.9 (+)N.VM.10 (+)N.VM.11 (+)N.VM.12	A.REI.5 A.REI.6a,b A.REI.7 (+) A.REI.8 (+) A.REI.9 A.REI.11 A.REI.12				A.CED.2 A.CED.3a		
Unit 8 • Conic Sections									G.GPE.1 G.GPE 2 (+) G.GPE.3 (+)

AP Course Descriptions & Standards

Currently, Hilliard City Schools offers three AP Math courses - AP Statistics, AP Calculus AB, and AP Calculus BC. Each course's Standards and Scope & Sequence are determined by the AP Central College Board. Links to each of these courses descriptions, including Standards, Scope & Sequence, and Exam Descriptions, are below:

AP Statistics: https://apcentral.collegeboard.org/courses/ap-statistics
AP Calculus AB: https://apcentral.collegeboard.org/courses/ap-calculus-ab
AP Calculus BC: https://apcentral.collegeboard.org/courses/ap-calculus-bc

AP Courses offered are determined annually through the Program of Studies process.

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Ready For Tomorrow

Phone: 614.921.7000

Website: hilliardschools.org

Address: 2140 Atlas St | Columbus, Ohio 43228

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